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Hegel's Conceptual Group Action on Creative Dynamics in Music

Was die Wahrheit ist, ist weder das Sein noch das Nichts, sondern dass das Sein in Nichts und das Nichts in Sein — nicht übergeht, sondern übergegangen ist.
Ihre Wahrheit ist also diese *Bewegung* des unmittelbaren Verschwindens des einen in dem anderen: das *Werden*.[1]

G. W. F. HEGEL, WISSENSCHAFT DER LOGIK, I.

1. Introduction

In *Musical Creativity*,[2] we have presented a model of musical creativity and given a number of examples, reaching from music theory to musical composition and music technology. We shall give a short description of the model's main components in the section 2.3 of this paper (entitled "The Conceptual Box Structure"). Although those examples confirm the validity of the model, there is one single component, where the model is still abstract and far from operational in terms of concrete actions to be taken. This component can be described using the common metaphor of the "box", which has to be opened in the creative process. In our model, the box is realized by what we call a "critical concept". The decisive step is then to identify the box's "walls" and to open them. We are fully aware that there is a major debate on creativity. Our position in this context can best be traced from our book, in particular from chapters 17, 18, 19, and 20. The present paper however focuses on a specific methodology of acting in a creative way, not on the general

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[1] GEORG WILHELM FRIEDRICH HEGEL, *Wissenschaft der Logik*, Frankfurt am Main, Suhrkamp, 1978[2] (G. W. F. Hegel: Werke in zwanzig Bänden, 5), I, p. 83 («The truth is neither being or nothing, but that being does not pass over, but has passed over into nothing, and nothing into being. Their truth is therefore this *movement* of the immediate vanishing of the one into the other: *becoming*», GEORG WILHELM FRIEDRICH HEGEL, *Hegel's Science of Logic*, trans. by Arthur Vincent Miller, London, Allen & Unwin – Amherst (NY), Humanities Press, 1969, p. 83).

[2] Cf. GUERINO MAZZOLA - JOOMI PARK - FLORIAN THALMANN, *Musical Creativity. Strategies and Tools in Composition and Improvisation*, Berlin – Heidelberg, Springer, 2011.

debate. This paper also does not claim to be a philosophical discourse, but we make use of a philosophical approach to delineate a general method to develop creativity in a very practical way.

This decisive step is what remains quite abstract in our previous work. The present paper presents a mechanism that is designed to catalyze that step by offering a concrete, but generic body of concepts and the action of a group of transformations of this body, a toolbox that should offer a set of operational perspectives onto the critical concept's walls. As was already pointed out in our book,[3] that the comprehension of such a mechanism for creativity is not a sufficient condition for effectively producing creative results, it is only meant to be a fairly important and useful procedure for creative actions.

We are deducing the conceptual body and the group action from the first paragraphs of Georg Wilhelm Friedrich Hegel's *Wissenschaft der Logik*.[4] This might be a logical approach since Hegel's initial dynamics in his logical architecture is in fact strongly related to the concept of "concept", i.e., to the basic structure of any concept. This is plausible since his incipit of thoughts claims to be the very beginning of the action of thinking, and in this moment, the very nature of conceptualization is at stake. We shall discuss Hegel's text and deduce the conceptual body we are proposing, a body built from six concepts which we for good reasons call *Hegel's body*, it is denoted by $\mathcal{B}$. The group that acts on $\mathcal{B}$ will be called the *Hegel group*, it is denoted by $\mathcal{H}$. We give a precise definition of these objects in sections 2.1 and 2.2 ("Hegel's Initial Thought Movement in *Wissenschaft der Logik*" and "The Implicit Group Structure").

But it is of course also highly problematic to deduce a mathematical structure such as a group action from Hegel's text since the basic situation of that incipit is far from being given a shape that could presuppose mathematical concepts for its description. This is correct, and we shall see that our mathematical concept framework is not requiring set theory, group theory and similar conceptual architectures. We simply use these concepts because they describe in their simplicity (which doesn't require the full mathematical formalism) what Hegel implies in his philosophical prose. Insofar, it is our claim that Hegel implicitly uses some very simple operations that our small group $\mathcal{H}$ (it is a Klein four-group) comprises. We leave it to philosophers to discuss our approach in terms of what they consider being a valid argument with regard to Hegel's thought dynamics. We are also

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[3]   Cf. *ibidem.*

[4]   Cf. G. W. F. Hegel, *Wissenschaft der Logik*, cit.

aware of the still problematic state of Hegel's text, a fact that has been discussed through the history of philosophy to the present.[5] However, we believe that our precise setup could help avoid those well-known rhetorical deformations of dialectic argumentation which has often generated nothing more than *ex post* circumlocution of results that had been generated by totally different methods.

It could be argued that we should also consider Hegel's writings on musical aesthetics. This paper is however not focusing on Hegel's ideas about music, we only use his very primordial ideas about how we start thinking. Logically speaking these ideas are independent of later developments in other Hegelian works. This is Hegel's own approach, whose very beginning of thoughts are set up in his initial sentences of *Wissenschaft der Logik*.[6]

Among the six basic concepts in the Hegel body, namely being, nothing, space, time, fact, and gesture, the latter plays a dominant role in what follows. Although we should not presuppose higher mathematical concepts here, we believe that it is advantageous to recall the precise definition of a gesture which we have given in the mathematical theory of gestures (in music),[7] since in the examples of this paper, this definition will be used.

A gesture refers to two structures, a directed graph Δ, the gesture's "skeleton", and a topological category X, the gesture's "body", see Figure 1. A gesture, denoted by $\delta : \Delta \to X$, is a map that assigns to every vertex v of Δ an object $\delta(v)$ in X, and to every arrow a of Δ a continuous functor $\delta(a) : \nabla \to X$ from the unit interval (category) $\nabla = (I, \leq)$, $I = [0, 1]$, to X in such a way that the configuration of arrows is conserved by the configuration of functors. This means that if, for example, the head y of arrow a is the tail of arrow b, then $\delta(a)(1) = \delta(b)(0)$.

5 See for example Aᴄʜɪᴍ Iʟᴄʜᴍᴀɴɴ, *Kritik der Übergänge zu den ersten Kategorien in Hegels "Wissenschaft der Logik"*, «Hegel Studien», XXVII (1992), pp. 11-25.

6 Cf. G. W. F. Hᴇɢᴇʟ, *Wissenschaft der Logik*, cit.

7 Cf. Gᴜᴇʀɪɴᴏ Mᴀᴢᴢᴏʟᴀ - Mᴏʀᴇɴᴏ Aɴᴅʀᴇᴀᴛᴛᴀ, *Diagrams, Gestures and Formulae in Music*, «Journal of Mathematics and Music: Mathematical and Computational Approaches to Music Theory, Analysis, Composition and Performance», I, 1 (2007), pp. 23-46; Gᴜᴇʀɪɴᴏ Mᴀᴢᴢᴏʟᴀ, *Categorical Gestures, the Diamond Conjecture, Lewin's Question, and the "Hammerklavier" Sonata*, «Journal of Mathematics and Music: Mathematical and Computational Approaches to Music Theory, Analysis, Composition and Performance», III, 1 (2009), pp. 31-58.
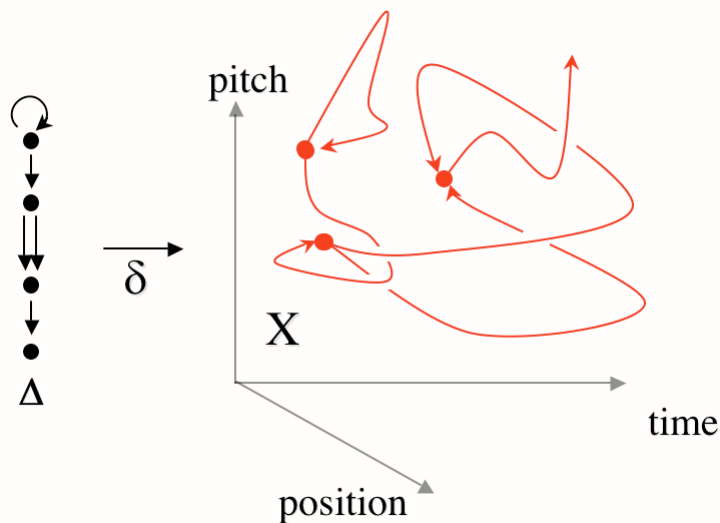
Fig. 1: The visual representation of a gesture whose body is a simple musical instrument space.

This paper is structured as follows: in section 2.2 ("The Implicit Group Structure"), we introduce and discuss the Hegel body $\mathcal{B}$ and the Hegel group $\mathcal{H}$. In section 3, "The $\mathcal{H}$ Action on the Yoneda Model of Creativity", we discuss the Hegel group action in the mathematical model of creativity that refers to Yoneda's lemma in category theory. Section 4, "The Hegel body $\mathcal{B}$ in the Concept Architecture of Forms and Denotators" is dedicated to a Hegelian interpretation of the concept architecture of forms and denotators which has been a backbone of mathematical music theory and its computerized implementation. Section 5, "The Usage of $\mathcal{H}$ for the Dynamics of Creativity", discusses the application of the Hegel group action for the understanding of creativity in musical compositions, such as the fanfare in Ludwig van Beethoven's *Hammerklavier* Sonata op. 106 and the incipit of Franz Liszt's *Mephisto Walzer*. Section 6 (entitled "An Experimental Composition") presents a small experimental composition created using the Hegel group action. Section 7 ("Still More Symmetries? Future Developments") shortly discusses the question concerning additional symmetries of the Hegel body. We should stress for non-mathematicians that a mathematical category is a different concept from a philosophical category.

2. The Hegel Concept Group $\mathcal{H}$

In this section we first analyze Hegel's initial thought movements in his *Wissenschaft der Logik*.[8] This will lead to a conceptual configuration built from six components, together with a group action, introducing what will be called the Hegel group $\mathcal{H}$. Our scope here is not to interfere with philosophical debates, but to elaborate a group structure that can be used in practical creative contexts. Nevertheless, we believe that the Hegel group structure could help understand some of the inherent dynamics in Hegel's primordial thoughts.

2.1 Hegel's Initial Thought Movement in „Wissenschaft der Logik"

All the following German quotations are taken from the modern German version of his writings,[9] also referenced in the online website Zeno.org.[10] The English quotations are taken from the Miller's translation of the Suhrkamp edition,[11] also referenced in the website marxist.org.[12]

The Hegelian system of logic starts with these words:

> A. Sein
> *Sein, reines Sein*, – ohne alle weitere Bestimmung. In seiner unbestimmten Unmittelbarkeit ist es nur sich selbst gleich und auch nicht ungleich gegen Anderes, hat keine Verschiedenheit innerhalb seiner noch nach aussen. Durch irgendeine Bestimmung oder Inhalt, der in ihm unterschieden oder wodurch es als unterschieden von einem Anderen gesetzt würde, würde es nicht in seiner Reinheit festgehalten. Es ist die reine Unbestimmtheit und Leere. Es ist *nichts* in ihm anzuschauen, wenn von Anschauen hier gesprochen werden kann; oder es ist nur dies reine, leere Anschauen selbst. Es ist ebensowenig etwas in ihm zu denken, oder es ist ebenso nur dies leere Denken. Das Sein, das unbestimmte Unmittelbare ist in der Tat *Nichts* und nicht mehr noch weniger als Nichts.[13]

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[8]   Cf. G. W. F. Hegel, *Wissenschaft der Logik*, cit., pp. 82-83.

[9]   Cf. *ibidem*.

[10]   Bibliothek Zeno, *Georg Wilhelm Friedrich Hegel. Wissenschaft der Logik*, http://bit.ly/1ueqV6p, Erstes Kapitel – Teil A, B, C, accessed November 15th, 2014.

[11]   G. W. F. Hegel, *Hegel's Science of Logic*, cit., pp. 82-83.

[12]   Marxist Internet Archive, *Hegel's Science of Logic*, http://bit.ly/1Hq1d9p, Volume 1 – Book 1, accessed December 3rd, 2014.

[13]   G. W. F. Hegel, *Wissenschaft der Logik*, cit., p. 82 («A. Being. *Being, pure being,* without any further determination. In its indeterminate immediacy it is equal only to itself. It is also not unequal relatively to an other; it has no diversity within itself nor any with a reference outwards. It would not be held fast in its purity if it contained any determination or content which could be distinguished in it or by which it could be distinguished from an other. It is pure indeterminateness and emptiness. There *is nothing* to be intuited in it, if one can speak here of intuiting; or, it is only this pure intuiting itself. Just as little is anything to be thought in it, or it is equally only this empty thinking. Being, the indeterminate immediate, is in fact *nothing,* and neither more nor less than *nothing*», G. W. F. Hegel, *Hegel's Science of Logic*, cit., p. 82).

The pure being is pure indeterminacy and emptiness. Emptiness is a spatial category, this is confirmed by the statement «to be thought in it», «it», the being. The nothingness is a spatial insight: penetrating pure being results in recognizing emptiness, nothingness. The preposition «in» is opposed to «out». Both refer to a boundary of a region that we cannot understand but in a spatial way. This spatial understanding is omnipresent in conceptual architectures, such as mathematical set theory or process theory.

Therefore, to the concept of being we have to add the concept of a generic space. Such a space cannot be the concrete physical space, at this stage it is a germ of spatiality, nothing more. But it is a conceptual component of being and nothingness. And it is not only a being out there, it is the action of thinking that reifies «being», it is neither object nor subject. These are categories to be introduced later in Hegel's system.

> B. NICHTS
> *Nichts, das reine Nichts*; es ist einfache Gleichheit mit sich selbst, vollkommene Leerheit, Bestimmungs- und Inhaltslosigkeit; Ununterschiedenheit in ihm selbst. Insofern Anschauen oder Denken hier erwähnt werden kann, so gilt es als ein Unterschied, ob etwas oder *nichts* angeschaut oder gedacht wird. Nichts Anschauen oder Denken hat also eine Bedeutung; beide werden unterschieden, so *ist* (existiert) Nichts in unserem Anschauen oder Denken; oder vielmehr ist es das leere Anschauen und Denken selbst und dasselbe leere Anschauen oder Denken als das reine Sein. Nichts ist somit dieselbe Bestimmung oder vielmehr Bestimmungslosigkeit und damit überhaupt dasselbe, was das reine *Sein* ist.[14]

The concept of nothingness initiates being complete emptiness, again a spatial component of nothingness, shared with being. Being was emptiness when penetrated in the thinking movement, whereas nothingness is emptiness from the beginning. The conception, «Anschauen oder Denken» or «intuition or thought», determines it and therefore generates its being, the empty thought of nothingness generates its being. Nothingness is a being, and in fact, because it is emptiness, it is the pure being where Hegel started his discourse. What is important here is that both, being and nothingness, are created from each other by a movement of thoughts. Reification of each one happens through a movement of thought. This will be made explicit the following paragraph in Hegel's text:

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[14] G. W. F. HEGEL, *Wissenschaft der Logik*, cit., pp. 82-83 («B. Nothing. *Nothing, pure nothing*: it is simply equality with itself, complete emptiness, absence of all determination and content–undifferentiatedness in itself. In so far as intuiting or thinking can be mentioned here, it counts as a distinction whether something or *nothing* is intuited or thought. To intuit or think nothing has, therefore, a meaning; both are distinguished and thus nothing *is* (exists) in our intuiting or thinking; or rather it is empty intuition and thought itself, and the same empty intuition or thought as pure being. Nothing is, therefore, the same determination, or rather absence of determination, and thus altogether the same as, pure *being*», G. W. F. HEGEL, *Hegel's Science of Logic*, cit., p. 83).

C. Werden
a. Einheit des Seins und Nichts
Das reine Sein und das reine Nichts ist also dasselbe. Was die Wahrheit ist, ist weder das Sein noch das Nichts, sondern dass das Sein in Nichts und das Nichts in Sein — nicht übergeht, sondern übergegangen ist. Aber ebensosehr ist die Wahrheit nicht ihre Ununterschiedenheit, sondern dass *sie nicht dasselbe*, dass sie *absolut unterschieden*, aber ebenso ungetrennt und untrennbar sind und unmittelbar *jedes in seinem Gegenteil verschwindet*. Ihre Wahrheit ist also diese *Bewegung* des unmittelbaren Verschwindens des einen in dem anderen: *das Werden*; eine Bewegung, worin beide unterschieden sind, aber durch einen Unterschied, der sich ebenso unmittelbar aufgelöst hat.[15]

Hegel starts with a seemingly contradictory statement: being and nothingness are the same. And in fact, he contradicts this statement some lines later, saying that «they are not the same». This contradiction can be resolved if we view Hegel's statements as assertions of aspects of being and nothingness, not of their full «truth». We might use a geometric metaphor to illustrate this understanding. If one is positioned on a Moebius strip, it is true that it has two sides, the one where one stands, and the opposed one. But one knows that a Moebius strip has only one side. The opposed side is just a part of the front side. This apparent contradiction is resolved when one realizes that the other side is a local statement, while the sameness of the two sides is a global statement: one may walk from the first to the second local side on a global trajectory.

In this sense, sameness of being and nothingness is a global statement, while their difference is a local one. Being and nothingness are two local aspects of the same global concept. Hegel offers a clear method to perform the trajectory between being and nothingness: it is the movement that was already alluded to in the previous paragraphs when Hegel described the movement between being and nothingness in the thinking action. Now, he makes this movement explicit: «Their truth is therefore, this movement of the immediate vanishing of the one into the other: becoming». With this, to the basic concepts of being, nothingness, and space, Hegel adds a next one: becoming. It is however a delicate conceptualization since it is that movement of thought that has no subject or object yet, it is a pure action. We therefore propose to rename this concept and to call it "gesture", a basic action that is not yet embedded in the dichotomy of object/subject.

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[15]   G. W. F. Hegel, *Wissenschaft der Logik*, cit., p. 83 («C. Becoming. (a) *Unity of Being and Nothing. Pure Being* and pure *nothing* are, therefore, the same. What is the truth is neither being nor nothing, but that being – does not pass over but has passed over – into nothing, and nothing into being. But it is equally true that they are not undistinguished from each other, that, on the contrary, they are not the same, that they are absolutely distinct, and yet that they are unseparated and inseparable and that each immediately *vanishes* in its *opposite*. Their truth is therefore, this movement of the immediate vanishing of the one into the other: *becoming*, a movement in which both are distinguished, but by a difference which has equally immediately resolved itself», G. W. F. Hegel, *Hegel's Science of Logic*, cit., p. 83).

In might seem that we have introduced the concept of a gesture in an arbitrary way. Let us make clear why this impression is wrong. The rationale for our conceptual choice was not to change Hegel's terminology, but first of all to solve the apparent (onto)logical contradiction between Being and Nothingness. Our discourse has in fact solved the contradiction by the introduction of a new concept (gesture) as described above. And it has done so *without* leaving classical logic in favor of some more exotic logics, such as intuitionistic, fuzzy or paraconsistent variants. Let us also remark that gestures are generically useful in artistic utterance, but see sections 5.3 ("Escher's Theorem for Beethoven's Fanfare in the *Hammerklavier* Sonata op. 106"), 5.4 ("The Rotation $S@N$ as a Driving Creative Force in the Incipit of Liszt's *Mephisto Walzer* No. 1"), and 6 ("An Experimental Composition").

The text is moreover also specific about an aspect that every action seems to embody: time. Hegel writes «but that being does not pass over but has passed over».[16] This reveals a time category where presence and past are distinguished. Thus, we have to add the concept of time to the space concept in Hegel's setup. Finally, the statement of «being passed over» specifies a further conceptual aspect, namely that after the gestural action is established, there is a resulting fact, the transition of being into nothingness, and *vice versa.*

Summarizing, we have collected a sixfold conceptual anatomy, grouped into three pairs of corresponding concepts: Being/Nothingness (S/N for German *Sein/Nichts*), Space/Time (R/Z for German *Raum/Zeit*), and Gesture/Fact (G/F for German *Geste/Faktum*). These six conceptual "elementary particles" are shown as vertices of an octahedron in Figure 2, we shall call them "fermions" in an allusion to elementary particle physics, where fermions are the particles that represent matter – as opposed to forces, which are represented by particles that are called "bosons". We are using the famous Borromean rings in this representation to indicate that these six elementary concepts are not independent from each other. They build an irreducible body of concepts, which we call the "Hegel body", and denote as a set (by abuse of language, since mathematical formalisms should not matter yet) by $\mathcal{B} = \{S, N, R, Z, G, F\}$.

2.2 The Implicit Group Structure

The evident symmetry of the Hegel body $\mathcal{B}$ is not by case, and it is not our invention, but results from Hegel's approach as discussed above. In fact, the crucial movement between S and N is defined by the gestural becoming, G, that transforms S into N and vice versa. In this movement, there is also the result in facticity F, that terminates the

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16   *Ibidem.*

movement and puts into its temporal past tense. This transformation may be interpreted as a symmetry of $\mathcal{B}$, namely the 180^0 rotation around the axis spanned by G and F, which we denote by $G@F$. In Figure 2, this axis corresponds to the question "HOW?", its rotational action answers the question of how S is transformed into N and vice versa, namely by the gestural action. Using our physical metaphor, the pair $G@F$ plays the role of a boson, a force particle that moves S into N and vice versa. This interpretation is remarkable since it gives to pairs of fermions a role of bosons. In other words: The Hegel body $\mathcal{B}$ is simultaneously a body of material and of forces. The fermions are moved by bosons, and the bosons are generated by pairs of fermions. This is a philosophically essential proposition as it suspends the question of what is more elementary: movement or moved things, this is another justification of the Borromean ring visualization.
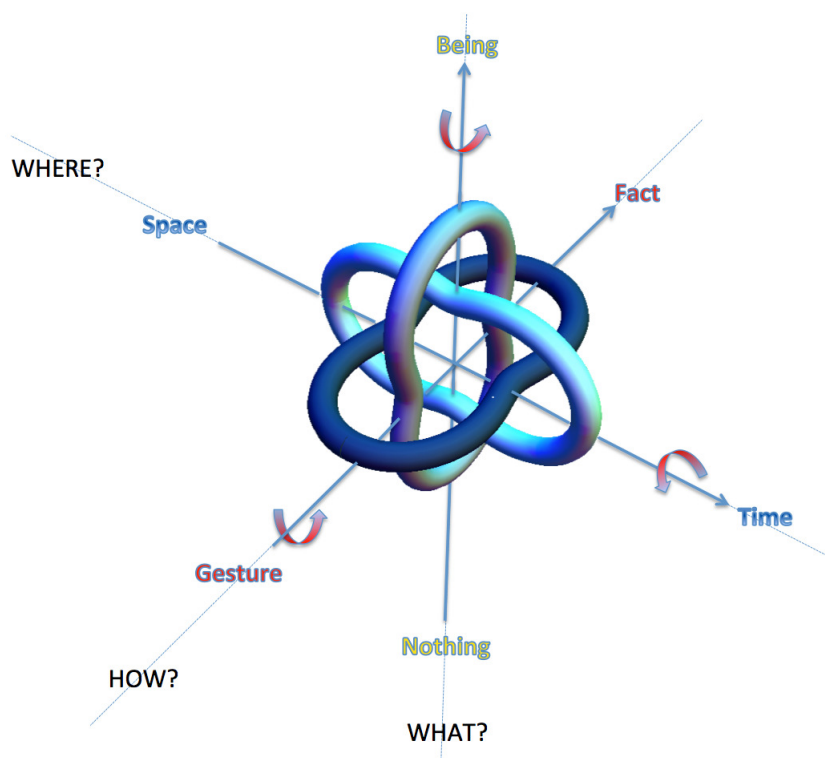

Fig. 2: Hegel's concept architecture, the Hegel body $\mathcal{B} = \{S, N, R, Z, G, F\}$, and the $\mathcal{H}$ group action, where the rotational axes answer the questions: how, what, where?

But let us first complete the transformational setup defined by the bosonic actions: we have three actions, each defined by a bosonic pair of fermions,

♦ the 180^0 rotation $G@F$ around the axis G—F, yielding the permutation $(SN)(RZ)$

♦ the 180^0 rotation $R@Z$ around the axis R—Z, yielding the permutation $(SN)(GF)$

♦ the 180^0 rotation $S@N$ around the axis S—N, yielding the permutation $(RZ)(GF)$

Together with the identity Id (all of $\mathcal{B}$ remains fixed), this defines a group
$$\mathcal{H} = \{Id, G@F, R@Z, S@N\}$$
of permutations of $\mathcal{B}$. It is evident that this commutative group verifies $x^2 = Id$ for all x, and $x \cdot y = z$ for any two different $x, y \neq Id$, and where z is the third non-identity. For example, $G@F \cdot R@Z = S@N$. This group is isomorphic to the Klein four-group K_4, but we stress the equal roles of all three generators. This group of permutations is called the "Hegel group". The orbits of the group's action are exactly the bosonic pairs.

The group structure extends the semantics of the original Hegel context, where only the action of $G@F$ on S, N are explicitly thematized. Let us therefore interpret the complete action
$$\mathcal{H} \times \mathcal{B} \to \mathcal{B}.$$

1. $G@F : S \leftrightsquigarrow N$

This is Hegel's original movement of becoming, which he later specifies into ceasing-to-be $S \rightsquigarrow N$ and coming-to-be $N \rightsquigarrow S$.

2. $G@F : R \leftrightsquigarrow Z$

The gestural operation maps time to space via the time-parametrization of a gesture (see also the mathematical theory of gestures developed in *Diagrams, gestures and formulae in music*).[17] The fact as a result of a gesture recreates the time that has produced the spatial points. Let us recall here a significant statement by the great mathematician Henri Poincaré in *La valeur de la science*:

> Localiser un objet, cela veut dire simplement se représenter les mouvements qu'il faudrait faire pour l'atteindre; [...] il ne s'agit pas de se représenter les mouvements eux-mêmes dans l'espace, mais uniquement de se représenter les sensations musculaires qui accompagnent ces mouvements et qui ne supposent pas la préexistence de la notion d'espace.[18]

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[17] Cf. G. Mazzola - M. Andreatta, *Diagrams, Gestures and Formulae in Music*, cit.

3. $R@Z : G \leftrightsquigarrow F$

The time as a generator of spatial points (points as results of the pricking gesture) is embodied in the gestural movement that creates its factual results. Conversely, facts as spatial localizations are only thought as endpoints of a time line of a gestural movement.

4. $R@Z : S \leftrightsquigarrow N$

Being as a thinking activity in time is annihilated to nothing when fixed in to spatial points. Conversely, points when rethought as results of the pointing action recover their temporal origin.

5. $S@N : G \leftrightsquigarrow F$

A gesture, when taken as a being, is transformed into its resulting fact which is the nothingness that terminates the gesture. Conversely, a fact as a nothingness, when thought as a result of an action, recovers its generative gesture.

6. $S@N : R \leftrightsquigarrow Z$

Time, as the movement of being (recall Hegel's becoming), when frozen to nothingness, generates spatial points, endpoints. Conversely, if a point in its nothingness (it has no inner substance, so to speak) is rethought of what it brings to be, its being recreates time where the point was moved to its present location. It seems adequate to recall here Gurnemanz's lesson to Parsifal: «Du siehst, mein Sohn, zum Raum wird hier die Zeit».[19]

It is evident that all these operations relate to each other, and this is essential, they are not independent concepts, but define the irreducibility of the Borromean architecture and, in fact, of the Hegelian setup.

Example 1: before we investigate more in depth the implications of this group action with regard to the creativity process, it may be useful to give a first elementary musical example of the $\mathcal{H}$ action. Let us look at the primitive action that a musician has to perform to produce a sound, hitting a key on the piano, singing a note, or blowing a tone on a trumpet. Such an action has three parts: the initial gesture moving out from the nothingness of silence to the sound production, then the sound as a product, as a being that has a factual reality, and third the termination of the sound production, the exiting gesture

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[18]   «To localize an object simply means to represent to oneself the movements that would be necessary to reach it; [...] it is not a question of representing the movements themselves in space, but solely of representing to oneself the muscular sensations which accompany these movements and which do not presuppose the preexistence of the notion of space», Henri Poincaré, *La Valeur de la Science*, Paris, Ernest Flammarion, 1905, pp. 87-88.

[19]   «You see, my son, here time becomes space».

taking back the fact to nothingness. This is exactly what the operation $R@Z = (SN)(GF)$ does, it permutes nothingness and being as well as gesture and fact. The operation $R@Z$ first maps $N \rightsquigarrow S$ and $G \rightsquigarrow F$, creating the factual being of the sound. Then, applied in the other direction, it takes back $S \rightsquigarrow N$ and $F \rightsquigarrow G$, this corresponds to the formula $(R@Z)^2 = Id$. The orbit of $R@Z$ is what the creation of a sound realizes in terms of Hegelian action.

2.3 The Conceptual Box Structure

The box structure of the Hegel body $\mathcal{B}$ is shown in Figure 3. The box (a cube) is the dual of the octahedron defined by the six conceptual components of $\mathcal{B}$. Each component corresponds to a wall of the box. This is a perfect visualization of the principles of the creative process which we have discussed in *Musical Creativity*.[20] Let us give a very short summary of such a process:

1. Exhibiting the open question
2. Identifying the semiotic context
3. Finding the question's critical sign or concept in the semiotic context
4. Identifying the concept's walls
5. Opening the walls
6. Displaying extended wall perspectives
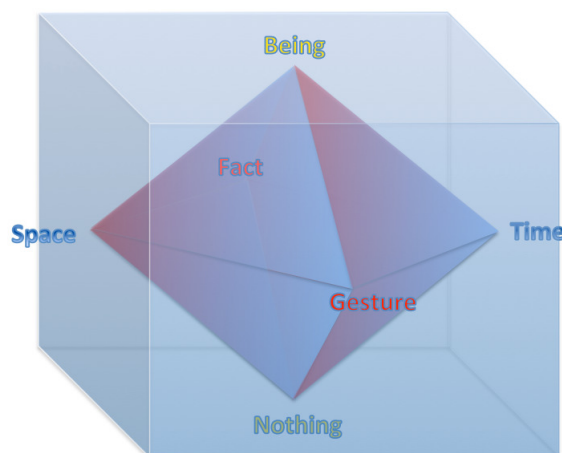7. Evaluating the extended walls

Fig. 3: The dual of Hegel's concept octahedron is a cube, whose six walls are associated with Hegel's basic concepts.

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[20]   Cf. G. Mazzola - J. Park - F. Thalmann, *Musical Creativity. Strategies and Tools in Composition and Improvisation*, cit.

The metaphor of a concept's walls is now perfectly realized by the box of Hegel's body, which we may call "Hegel's box". This coincidence is what we shall take as the starting point of our approach to creativity, namely that the critical action of identifying a concept's walls is made concrete by Hegel's box. This means that we claim that the Hegel action $\mathcal{H} \times \mathcal{B} \to \mathcal{B}$ enables a machinery that helps identify a concept's walls.

To realize this plan we however have to understand why the action $\mathcal{H} \times \mathcal{B} \to \mathcal{B}$ is a tool that could help understand and eventually soften a critical concept's walls. The first observation to this end is that in Hegel's approach, when discussing being and nothingness, he deals with a conceptual architecture that is extremely elementary. In his words, it is about emptiness, about the very beginning of conceptual thinking. This means that what he proposes, and what we have drawn from his idea, is a conceptual framework that is not yet loaded with any specific architectural details, it is "the empty canvas of conceptual construction" that we see in Hegel's text. This is the rationale that motivates us to use this conceptual canvas to investigate any critical concept in a creative process's crucial step of wall identification.

This means that using the Hegel action in the analysis of a critical concept, one might be able to identify its walls. What we called walls in the theory of creativity[21] are the concept's properties, characteristics, and specificities that circumscribe it in a more or less explicit form. In other words, what defines its inner structure, but also, what delimits it from other concepts, what it is not. Therefore, using the very basics of a concept's conception (yes, this is circular, but this is essential in the beginning of conceptual organi-zation), one should get a supporting machinery dealing with the identification process of walls.

In what follows we will investigate the Hegel action from two perspectives: first, we reconsider the mathematical model of creativity as described in *Musical Creativity*[22] and in the article *Towards a Categorical Theory of Creativity for Music, Discourse, and Cognition*.[23] Second, we analyze the architecture of forms and denotators, which have played a major role in the conceptual framework of mathematical music theory, as shown in *The Topos of Music*.[24]

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[21] Cf. *ibidem.*

[22] Cf. *ibidem*, section 19.2.

[23] Cf. Guerino Mazzola [et al.], *Towards a Categorical Theory of Creativity for Music, Discourse, and Cognition*, in *Mathematics and Computation in Music. 4th International Conference, MCM 2013*, ed. by Jason Yust, Jonathan Wild and John Ashley Burgoyne, Heidelberg, Springer, 2013.

[24] Cf. Guerino Mazzola, *The Topos of Music. Geometric Logic of Concepts, Theory, and Performance*, in collaboration with Stefan Göller and Stefan Müller, Basel [etc.], Birkhäuser, 2002, II, section 6.

3. The $\mathcal{H}$ Action on the Yoneda Model of Creativity

In category theory, Yoneda's idea was to define a functor $Yon_{\mathcal{C}} : \mathcal{C} \to \mathcal{C}^{@}$, where $\mathcal{C}^{@}$ is the category of set-valued presheaves over the category $\mathcal{C}$, by assigning to each object A of $\mathcal{C}$ a presheaf $@A : \mathcal{C}^{opp} \to$ **Sets** defined by $@A(X) = X@A(=\mathcal{C}(X, A))$ and for each morphism $f : A \to B$ in $\mathcal{C}$ a natural transformation $@f : @A \to @B$ given by $@f(X) : X@A \to X@B : g \mapsto f \circ g$. Yoneda's lemma says that $Nat(@A, F) \xrightarrow{\sim} F(A) =: A@F$, for every object A of $\mathcal{C}$ and every functor F in $\mathcal{C}^{@}$. This means in particular for $F = @B$ that A and B are isomorphic if and only if their functors $@A$ and $@B$ are so. We may therefore replace the category $\mathcal{C}$ by its Yoneda-image in $\mathcal{C}^{@}$.

Although Yoneda's lemma enables the replacement of a given category $\mathcal{C}$ by its Yoneda-image in $\mathcal{C}^{@}$, the functor $@A$ must be evaluated on the entire category $\mathcal{C}$ to yield the necessary information for its identity. The creative moment comes in here: could we not find a subcategory $\mathcal{A} \subset \mathcal{C}$ such that the functor $Yon_{|\mathcal{A}} : \mathcal{C} \to \mathcal{A}^{@} : A \mapsto @A|_{\mathcal{A}^{opp}}$ is still fully faithful? We call such a subcategory "creative", and it is a major task in category theory to find creative categories which are as small as possible. One may even hope to find what we call an "objectively creative subcategory" for a given object A in $\mathcal{C}$, namely a creative subcategory $\mathcal{A}$ such that for this given object A in $\mathcal{C}$ there is a creative diagram D_A in $\mathcal{A}$ whose colimit C is isomorphic to A. Intuitively, a colimit of a diagram of spaces is obtained by gluing them along common subspaces; it is a generalized union operator. Taking a colimit is a natural condition since the functor $@A$ defines a big diagram whose arrows are the triples $(f : X \to Y, x \in X@A, y \in Y@A)$ with $y \circ f = x$. The colimit object C of such a diagram would ideally replace the functor $@A$ by a unique isomorphism from C to A.

In the context of the Yoneda Lemma with its creative subcategories, the generic model of creativity described in the section "The Conceptual Box Structure" (section 2.3) looks as follows:

1. Exhibiting the open question: understand the object A
2. Identifying the semiotic context: this is the category $\mathcal{C}$ where A has been identified
3. Finding the question's critical sign or concept in the semiotic context: this is A
4. Identifying the concept's walls: this is the uncontrolled behavior of $@A$
5. Opening the walls: finding an objectively creative subcategory $\mathcal{A}$
6. Displaying extended wall perspectives: calculate the colimit C of a creative diagram
7. Evaluating the extended walls: try to understand A via the isomorphism $C \xrightarrow{\sim} A$

Here is the correspondence between the Yoneda setup of creativity and the Hegel body: the paring of *G/F* corresponds to the pairing object functor *@X/X*. The object is a fact, an abstract point in the category *C*. The corresponding functor *@X* enriches the factual object by the entire system of arrows that are gestural pointers to *X*. The bosonic action *R@Z* maps *F* to *G*, i.e., *X* to *@X*. Moreover, it maps *S* to *N* in the sense that it switches from the object's identity, its simple being *S*, to its nothingness, its negation in the category's outside: all other objects that become the domains of the functor's arrows to *X*. On the other hand, the movement from *@X* to the colimit *C* takes the gestural aspect back to its factual reduction, to an object *C*. Finally, the negation *N* comes back to *S* as a negation of the negation, the object *C* that re-instanciates *X* form its negational functor.

4. The Hegel body $\mathcal{B}$ in the Concept Architecture of Forms and Denotators

Before we discuss the form and denotator concept architecture it is important to trace back this framework to the epistemological roots which define it as an application of the semiotic concept architecture set up by D'Alembert and Diderot in their *Encyclopédie*.[25] In their approach, an encyclopedia must comprise three characteristics: unity, completeness, and discoursivity. This means that it must realize a philosophical principle of unified presentation of knowledge, it must represent all knowledge (what we expect from a dictionary), and it must enable a discourse, a relational setup to compare its instances, the latter being given by the lexicographic ordering in a dictionary. This triple characteristic was interpreted in our form and denotator architecture by the following three characteristics: unity was realized by the principle that concepts refer to other concepts, a recursive typology. Completeness was realized by a complete set of types of references. Discoursivity was realized by linear order and recombination of concepts.

Given these principles, a denotator is a conceptual instance in a space, called form. We refer to *The Topos of Music*[26] for details and just recall the relevant features here. A denotator has coordinates, i.e., denotators to whom it refers, much as its form has coordinator forms to which it also refers. This is the recursive principle. It is the gestural aspect of this architecture. Moreover, the spatial aspect is covered by the referential typology: in topos

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[25] Cf. Sylvan Auroux, *La sémiotique des encyclopédistes: essaye d'épistémologie historique des sciences du langage*, Paris, Payot, 1979; Jean Baptiste Le Ronde D'Alembert, *Discours préliminaire des Editeurs*, Paris, 1751 (Encyclopédie ou Dictionnaire raisonné des sciences, des arts et des métiers, I); G. Mazzola, *The Topos of Music. Geometric Logic of Concepts, Theory, and Performance*, cit., II, § 6.1.

[26] Cf. G. Mazzola, *The Topos of Music*, cit.

theory, it is the three basic space types of limit, colimit, and powerset. Time is realized in the trajectory of gestures you have to perform to reach a denotator's recursive ingredients. Facticity is obtained when you reach the leaves of the denotator's (and the form's) recursive tree. Being is realized by the denotator's entire instance, while nothingness is realized by the linear ordering within the denotator system: the relation to what a denotator (or form) is not, what comes before and what comes after that instance.

This makes evident that the denotator and form concept architecture, which is the most general existing approach to precise conceptualization, and which has been implemented in music software with great success,[27] complies perfectly with the Hegel body $\mathcal{B}$. It is an open question to understand the Hegel action $\mathcal{H}$ in this situation. However, the exchange of time and space could be realized using the equivalence of breadth-first and depth-first search in forms that are built from iterated limits.

5. The Usage of $\mathcal{H}$ for the Dynamics of Creativity

The usage of the Hegel action for the dynamics of creativity is a multiple one. On the one hand, we can conceive it as a diagnostic tool without necessarily acting as a generator of creative extensions. On the other hand, it can be thought as a machine (though not a dead algorithm) that enables creative extensions. We want to discuss these two directions which, of course, are not exclusive: a good diagnosis can induce a creative extension, and the extensional spectrum can reveal quite a bit about the "patient's" health.

5.1 Two Preliminary Examples

Example 2. Let us give a first example of a diagnostic functionality of the Hegel action: Einstein's invention of a multiple time concept. Within our creative process scheme as displayed in section 2.3 ("The Conceptual Box Structure"), the critical concept is physical "time", time in the semiotic context of physics — not the primordial time concept which is part of the Hegel body. Let us take this concept as it was given before Einstein's in(ter)vention. When we inspect the walls of S and of N, it turns out that this concept is a singleton, it has no other copies out there, i.e., its non-being N is empty. In terms of grammar it is a singular being. Taking time and throwing it out to N by the transformation $G@F$ yields nothingness. This diagnosis generates the question "Why only one time"? Is there a physical reason for supporting this singularity of the concept? And Einstein's

27 See Gerard Milmeister, *The Rubato Composer Music Software: Component-Based Implementation of a Functorial Concept Architecture*, Heidelberg, Springer, 2009.

answer was that physical time could exist in a multiplicity of times, one for every inertial system, and that the gesture of throwing one such time instances to its nothingness in another inertial system would be realized through the famous Lorentz transformation.

Example 3. A second diagnostic example is the invention of the 3M Post-It, a creative process that has been analyzed in detail in the fourth chapter of *Musical Creativity*.[28] Here the critical concept is "adhesive"; its inventor, 3M chemist Dr. Spencer Silver, had created a substance that did not glue as required, but only "half of it". This time the gestural wall will be inspected: what are the concept's components (recall the referential characteristic of concepts discussed in section 4 – "The Hegel body $\mathcal{B}$ in the Concept Architecture of Forms and Denotators" –, as suggested by D'Alembert and Diderot)? One of them is that an adhesive must glue. This gluing concept's architecture in turn has a way of being that in its becoming has no further reference: it is a final fact, i.e., either gluing or not gluing, *tertium non datur*. This was exactly the point of the diagnosis that Dr. Silver learned from his friend Arthur Fry: there is no deeper reason to terminate the concept's reference tree on that final "glue" as opposed to "not glue". Gluing by 50% was introduced as a deeper conceptual reference: gluing with a percentage. The commercial success of this new adhesive proved that this creative conceptual extension was the right thing to do. In terms of denotator theory, the conceptual component of gluing that was given as a Boolean value was replaced by a real number value.

Evidently, the present state of the art is far from what one could coin an expert system. But this is not surprising since the full meaning of the Hegel action must be elaborated with respect to a variety of semiotic contexts for creative processes. However, it seems evident that a number of core questions around a concept can be built to open conceptual walls more easily.

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[28] Cf. G. Mazzola - J. Park - F. Thalmann, *Musical Creativity. Strategies and Tools in Composition and Improvisation*, cit., pp. 17-19.

5.2 The Challenge: Creating a Spectrum of Conceptual Extensions

In *Musical Creativity*,[29] creativity has been described as a process that takes place in a specific semiotic context. And the result of such a process is viewed as an extension of the given semiotic body. Creativity adds expression, signification and content. It is not a formal combinatorial game. Such an extension entails several critical aspects:

1. It need not to be a successful extension. For example, adding a color to mathematical symbols would very probably not solve any mathematical problem. Therefore the semiotic extension might be useless for the time being, but, in the long run, it might turn out to be a good move. This means that creativity should also be judged in the global perspective of the evolution of a semiotic system. This resembles biological evolution, where a local change might show its advantage or disadvantage only after a longer period of further evolution.

2. The conceptual extension, following the Hegel action, say, need not be unique. Opening walls might create an entire "spectrum" of conceptual extension which need not contradict each other. For example, the recent extension of counterpoint theory as described in *Computational Counterpoint Worlds* contains a variety of conceptual extension of what are consonances and dissonances within the 12-chromatic pitch class system, but simultaneously extends to microtonal pitch systems.[30]

3. Applying the Hegel action to a critical concept C is a manifold endeavor. For every one X of the six walls, one may create a conceptual extension $C(X)$. If one applies several extensions in a certain order, $C(X_1, X_2, \ldots X_n)$, say, it is probably not true that another extension following a permuted order, $C(X_{\pi(1)}, X_{\pi(2)}, \ldots X_{\pi(n)})$, would yield the same result.

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[29] Cf. *ibidem*.

[30] Cf. Octavio Alberto Agustin-Aquino - Julien Junod - Guerino Mazzola, *Computational Counterpoint Worlds*, Heidelberg, Springer, 2015.

Despite these general questions we should present a more concrete example of a conceptual spectrum created following the Hegel action. Our example relates to the operation $S@N = (RZ)(GF)$. In mathematical gesture theory,[31] one considers hypergestures, i.e., gestures $h : \Gamma \to \Delta\overrightarrow{@}X$ starting at the digraph Γ (their *skeleton*) and targeting to the topological category $\Delta\overrightarrow{@}X$ of all gestures of *skeleton* Δ which target to the topological category *X*. The Escher theorem[32] then states that we have an isomorphism of topological categories Escher: $\Gamma\overrightarrow{@}\Delta\overrightarrow{@}X \xrightarrow{\sim} \Delta\overrightarrow{@}\Gamma\overrightarrow{@}X$. This means that we may exchange the roles of the two *skeleta* Γ, Δ. In other words: the gesture h which maps curve parameters to gestures *qua* points in $\Delta\overrightarrow{@}X$ can be reinterpreted as a gesture where the points now become gestures and vice versa. This is exactly what the symmetry $S@N$ does: it exchanges G and F. And it also exchanges R and Z, which in the Escher setup makes sense since the space of gestural facts in $\Delta\overrightarrow{@}X$ is transformed into the time parametrization of gestures of $\Delta\overrightarrow{@}\Gamma\overrightarrow{@}X$. The Escher procedure therefore enables us to reinterpret gestures within concepts in permuted ways, and thereby create new meanings. For example, a hypergesture defined by a line of circles can be reinterpreted as a hypergesture defined by a circle of lines. This can yield a completely new understanding of a given concept. In music theory, first species counterpoint can be viewed as a circle that connects (within the pitch class circle) the line of the *cantus firmus* to the line of *discantus*. But the Escher permutation of roles would view this counterpoint as a time line of intervals, and therefore as a completely different understanding of what counterpoint means.

The Escher theorem is an extremely explicit key to the Hegel action using the $S@N$ operation. It would be interesting to search for Escher-like theorems which relate to the other two Hegel actions.

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[31] Cf. G. Mazzola - M. Andreatta, *Diagrams, Gestures and Formulae in Music*, cit.; G. Mazzola, *Categorical Gestures, the Diamond Conjecture, Lewin's Question, and the "Hammerklavier" Sonata*, cit.

[32] Cf. *ibidem*, section 2.4.

5.3 *Escher's Theorem for Beethoven's Fanfare in the* Hammerklavier *Sonata op. 106*
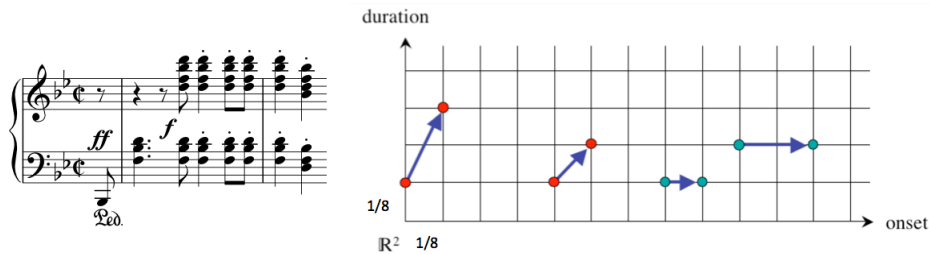

Fig. 4: The initial fanfare in Beethoven's *Hammerklavier* sonata op. 106.

We want to illustrate the creative movement as it can be interpreted using the above Escher technique as an expression of the *S@N* operation with a concrete example: the initial fanfare of Beethoven's *Hammerklavier* sonata op. 106, see Figure 4. The fanfare's rhythmical structure is shown to the right, it consists of eight points in the plane of onset and duration. The first gesture is a line from *A* to *B*, a halting gesture $A \rightarrow B$, see Figure 5.

In a hypergesture ρ, this gesture is mapped to the repeated halting gesture $\rho(A) \rightarrow \rho(B)$. The hypergesture ρ connects to a repetition of a halting gesture. To this moment, the fanfare is nothing but a not so creative confirmation of the initial gesture's being, namely the movement of $A \rightarrow B$ to $\rho(A) \rightarrow \rho(B)$. The creative action takes place when Beethoven's construction exchanges the gestural and the factual roles, following the methodology described above. The factual $A \rightarrow B$ becomes the gestural part in ρ', whereas the gestural movements $A \rightarrow \rho(A)$ and $B \rightarrow \rho(B)$ become the factual parts. This is the Escher isomorphism, applied to ρ, i.e. $\rho' = \text{Escher}(\rho)$. A simple topological deformation generates the second hypergesture σ. Observe that this latter hypergesture cannot be generated by direct deformation of ρ since it has a different orientation.

This Hegelian action exchanges the roles of duration and onset in the sense that a repetition of a halting gestures becomes the halting of a repetition gesture. This truly Escherian flipping movement gives the fanfare its full power. Although this example is a microscopic structure, it proves that creativity can have its germinal force in elementary compositional structures. We have shown in our article *Categorical gestures, the diamond*

conjecture, Lewin's question, and the "Hammerklavier" sonata[33] that this construction is not an isolated creation, but does shape the structure of the dramatic modulation E flat major ⤳ D major/B minor, mm. 189-197 of the first movement.

5.4 *The Rotation S@N as a Driving Creative Force in the Incipit of Liszt's* Mephisto Walzer *No. 1*

An example of the rotation $S@N = (RZ)(GF)$ can be found in the structure of the *Mephisto Walzer* No. 1 by Franz Liszt. The beginning of this composition presents a gesture of harmonic enrichment by the addition of fifths that attributes a harmonic role to the initial musical figure (a repeated tone E introduced by an acciaccatura D sharp). The pedal note, E, is followed by a sequence of concatenated fifths in an accelerated rhythm that highlights this process. Moreover, a second compositional gesture transforms the first theme into a second one, an intensely used sequence that develops within the score.
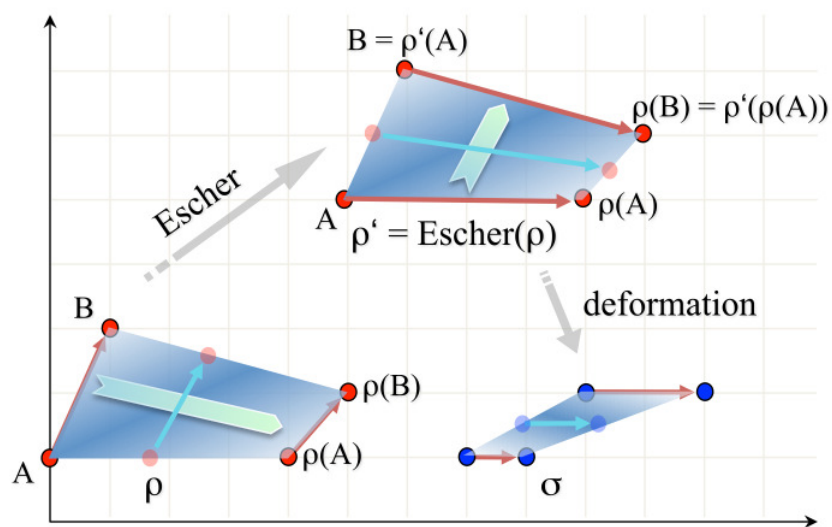
Fig. 5: The initial rhythmical hypergesture ρ of the fanfare is transformed into the hypergesture ρ′ via the Escher isomorphism, and then deformed into the target hypergesture σ of the fanfare.

In this composition, rhythmical and harmonic movements are linked in an insep-arable way. In the first bar we have the silence, a kind of mental preparation of the initial gesture. In the Hegel group interpretation, this silence could be viewed as nothingness before being. Nothingness becomes then an integral part of the artistic work.

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[33]   Cf. *ibidem,* § 5.4.

The initial gesture consists of an *ostinato* rhythm γ = D sharp-E-E-E, where the last two E notes are played as an echo of the initial one, see Figure 6. The rhythm is presented firstly in the extended form γ_0 = D sharp-E-E-E-E-E-E. In measures 4-5 we have the reduced sequence γ = D sharp-E-E-E, obtained from γ_0 by an operator of horizontal reduction, $h : \gamma_0 \longmapsto h(\gamma_0) = \gamma$. The following measures 6, 10, and 12, present the *ostinato* γ_0, together with its second half γ' = E-E-E (a second horizontal reduction γ' = E-E-E = $h'(\gamma_0)$) on a superposition of fifths: B, F sharp, C sharp. This kind of harmonic extension results from an operator of vertical completion of the fifth sequence. Measure by measure, we have a concatenation of sequence γ_0, γ, and γ'. The "factual" harmonic fifth sequence in the pitch class *space* $\mathbb{Z}_{12}$ is deployed as a "time gesture" by the *ostinato* rhythm. Therefore the first part of the creative process consists of an extension of the initial gesture in time, contextualizing the harmonic environment in the gesture γ. This realizes the Hegel transformation $S@N = (FG)(RZ)$.
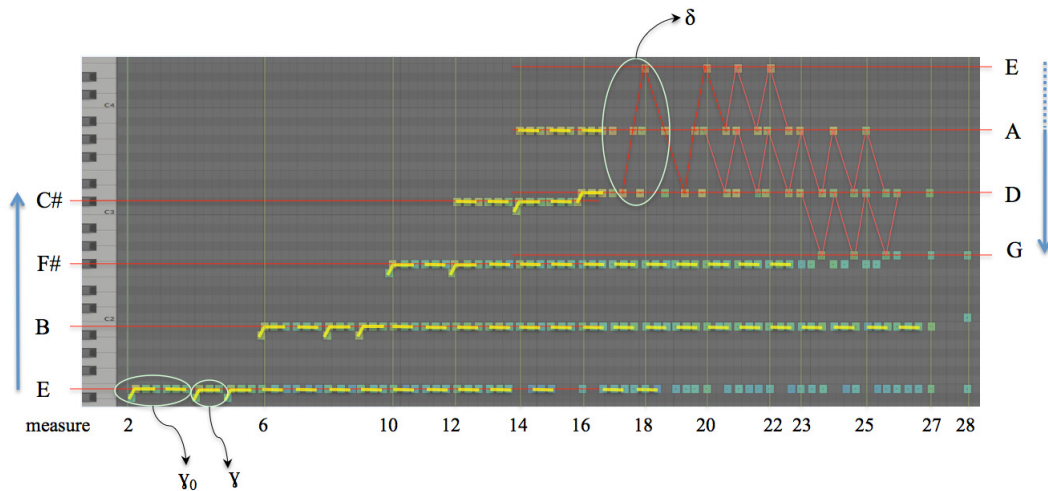

Fig. 6: A piano roll representation of mm. 2-28 of Liszt's *Mephisto Walzer* No. 1. Measures 1 and 29 are omitted since they are both tacet and surround the music by a nothingness of silence.

Let us shortly discuss the harmonic display here. Recall that diatonic scales are defined using six consecutive fifths. This harmonic structure is a spatial one in the space $\mathbb{Z}_{12}$ of pitch classes. Although a sequence of fifths has already a temporal potentiality, the ana-lyzed movement is still spatial (e.g. $T^7(E) = A$, where $T^7(x) = x+7$ is the fifth transposition). In our case, the measures 2-14 show a first ascending fifth sequence E-B-F sharp-C sharp, followed by a second descending sequence A-D-G, yielding a complete total of G-D-A-E-B-F

sharp-C sharp that defines D major. Liszt has succeeded in defining the harmonic basis of D major in a rhythmically triggered gesture. In measure 18, the chord with dynamics «*f marcato*» is composed as a symmetrical part around E, i.e. D-A-E-B-F sharp. The couple E-E is then the inferior and superior limit of this chord. This again confirms the inversion symmetry of D major around E.

Following the key signature (three sharps), the pedal note E could appear as the dominant of A major. However, this is not the case, since A major does not appear until m. 111. In mm. 2-27, E plays the role of second degree of D major. Then, in turn, in m. 28 D appears as the second degree of C major, in fact, almost unexpectedly appears a C in m. 28 that is confirmed in the following measures 30-34. The B of the left hand (m. 27) moves towards C (m. 28) as a leading tone. This movement suggests the effect of a "ghost note" to the superior melodic line: $D \rightarrow C$, see Figure 7. Summarizing, after the long preparation $E \rightarrow D$ as II $\rightarrow$ I in D major, we have another interpretation (this time virtual) II $\rightarrow$ I as $D \rightarrow C$. Tone E has still a relevant role for C major, being the symmetric pitch of the tonic with respect to the C major inversion symmetry around D, and it continues to act as a pedal in the reprise of the initial gesture (m. 35).

Fig. 7: The ghost note C in m. 28.

In mm. 17-18, the rhythmical and harmonic developments are completed by a "melodic creation". It is the birth of a melodic motive δ = D-A-E-A (with an initial rest rhythm, and a sequence of quaver-quaver-crochet-quaver). In Figure 6 the notes of δ and its rarefaction are highlighted by red lines. Mm. 20-26 present a progressive rarefaction of this fragment. Gesture δ will be then modified by transposition and variation of intervals in mm. 93-94, generating δ_1, see Figure 8. In m. 97 we will have a fusion of the head of γ (D sharp-E) and the tail of δ_1. This hybrid is followed by a new sequence of fifths, where E is effectively the dominant of A. A new version of δ, named δ_2, affirms clearly the tonality of A major.
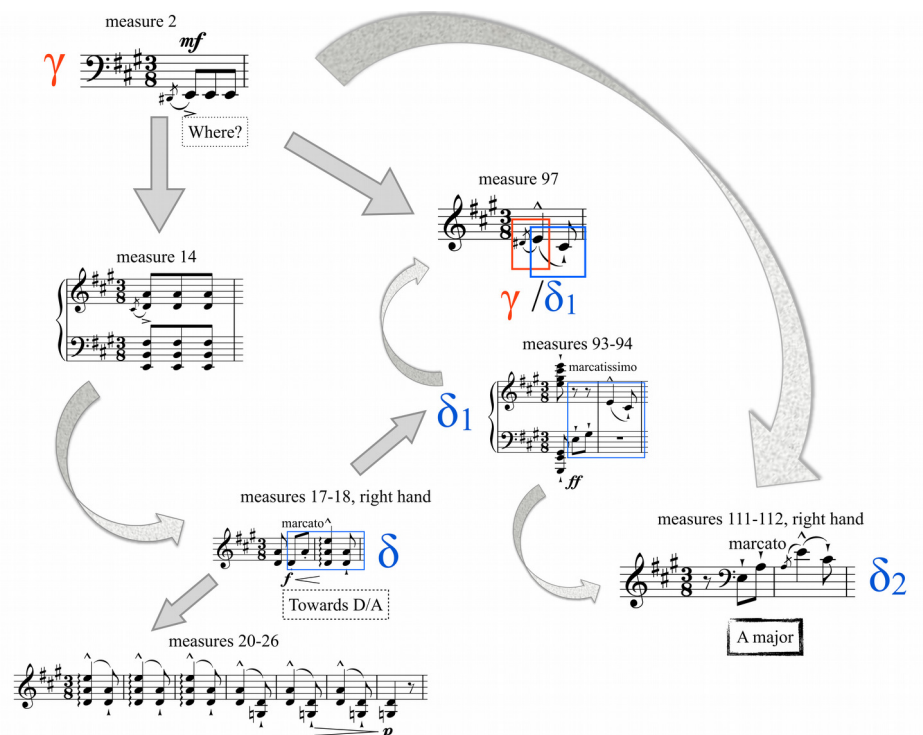
Fig. 8: In the *Mephisto Walzer* by Franz Liszt, the first gesture γ, is progressively repeated and translated in superposed fifths. The main theme δ, comes from these fifths played this time melodically. A transition gesture with the head of γ and the tail of δ leads to δ_1 and then to δ_2, the varied theme in A major.

Finally, in m. 29 we have the silence, symmetrically with the silence in m. 1. It is a gesture from the nothing to the being, and back to the nothing of silence, full of potentiality and tension. Rarefaction of tones corresponds to a dilation of rhythmical figures.

Summarizing, in the incipit of *Mephisto Walzer*, two creative gestures are evident:

1. rhythmic-harmonic deployment of γ_0 and its two parts γ, γ' in the circle of fifths of D major,

2. birth of δ through a deforming gesture of γ within its repositioning on the circle of fifths (rhythmic/harmonic variation).

Thus, the rhythmic structure unfolds through time the spatial relations of harmony. The factuality of abstract points is unfolded in a gestural transformation in time, realizing the Hegelian creative action $S@N = (RZ)(GF)$. The rhythm is the generative force of the

transformations $R \rightarrow Z$, $F \rightarrow G$. The action of deforming γ into δ is a movement generated by the rhythm and by the mapping of the circle of fifths into time.

6. An Experimental Composition

Let us try to compose a little musical piece using the Hegel model. It will be obtained, via *S@N* transformations, a little melody from an elementary gesture, see Figure 9.

Fig. 9: The little composition *The Hegel Song* has been composed starting from an elementary gesture and its variations constructed via the rotation *S@N = (RZ)(GF)*. This composition shows that fundamental compositional processes, variation and thematic development, can be easily obtained using the Hegel action.

The creativity model that we have defined can potentially describe all mechanism of artwork production. In the previous examples, the interest of analyzing pages of Beethoven and Liszt was given by the evidence of the construction principle. In these examples is not only evident the artistic "fact" but also the path of its creation. *Mephisto Walzer* does not start with the fifths already superposed and with the δ theme that dominates, but Liszt prepared the entry of the main theme revealing the very mechanism of construction.

One could observe that, sometimes, the artist can produce an artwork without making all steps explicit: it is the case of "sudden inspiration". This case also complies with

our creativity model since inspiration could be correlated to a time compression of steps within the Hegel scheme. These steps, even reduced to instants, must be defined.

A score contains a musical fact. Musicians' educated hands can realize such a fact via appropriate gestures. Playing fingers are like a dancer, which moves from one point to another: dance is not characterized by the targeted points, but by the trajectory to reach them. Recall Poincaré's citation in section 2.2 ("The Implicit Group Structure"). Gestures define the dynamics to reach these points. The choice of time and space (meter/tempo and keys), explicit in the musical score, contains all the required information to allow the hands' movements. Then, if we want to create a different musical fact, we may deform the used gesture, i.e., connecting it through a hypergestural line to a new gesture. A way to deform a gesture is by changing meter/tempo (Z) and notes (R). Time is the driving force, while notes are the "target" and thus the factual endpoints of any gesture. If a gesture terminates in an endpoint, the following gesture – that germinates from the preceding one at this endpoint – is connected to it via a hypergesture relation.

In our little example of a composition, realized using the Hegel group scheme, we will use meter/tempo and pitch indications to allow the pianist to create and annihilate the small beings commonly called "musical phrases". It is true that a lot of music is written with meter/tempo and pitch; however, in this case we intentionally start from a primitive gesture, named γ, without any precise meter/tempo indication, to represent the zero level of the pianist's action. A sequence of rotations around $S - N$ axis will generate the permutations (RZ) and (GF), and we will obtain a little melody.

In the case of a pianist – as already mentioned in the discussion of Liszt's *Mephisto Walzer*, where two notes are hit in the beginning – the primitive gesture here, γ, is a simple hitting of a key with one finger. This maps the spatial fact of a note symbol to a gesture in time. Repeating the same note means doubling this elementary gesture: 2γ. This repetition expresses time in a simple movement with a new characteristic of the note gesture as opposed to the timeless note symbol: γ can be positioned at different times. Some repetitions of γ and 2γ yield $n\gamma$. Until now, there is no quantification of time, and the gesture is localized on the same key. This unarticulated sequence of gestures must be connected and modified in order to generate a meaningful musical composition. Connected structures in music is realized via connected gestures. The same gesture, if repeated at regular intervals (time Z, in a 2/4 meter in our example) and shifted in particular points (space R), leads us

some musical facts, different but related by a common origin. This yields the step $n\gamma \longmapsto n\gamma(t)$; pitch space is deployed along time.

Until now, we have only couple of quavers; by doubling gestures we obtain a group of four notes, γ^2. In order to enrich the musical discourse, we can deform this simple scheme of four repeated notes. A different choice of spatial endpoints, a R transformation, deforming the hand gesture form, modifies the ribattuto notes into a more articulated sequence; thus $R : \gamma^2 \longmapsto \tilde{\gamma}^2$. For example, the sequence G-G-G-G becomes C sharp-B-D-B. Such sequences are typical for Bach's keyboard compositions; perhaps its simplicity and universality derives from these simple deformations of primitive gestures.

The Hegel Song

Fig. 10: The experimental composition *The Hegel Song*, created by Maria Mannone. Black: γ^{3-}, red: γ^4, blue: γ^{4+}, green: contour preserving deformation, yellow: metrical change $3/8 \longmapsto 6/8$.

In Figure 9, γ^3 is obtained from $\tilde{\gamma}^2$ by time modification $4/4 \longmapsto 3/4$, and $\tilde{\gamma}^{3-}$ from $\tilde{\gamma}^3$ by the suppression of the second note of each couple, realized by a "jumping" hand gesture. Transition t from $\tilde{\gamma}^{3-}$ to γ^4 is again a change of meter $3/4 \longmapsto 3/8$. Transition from γ^4 to γ^{4+} is a transformation of time (rests) to space, filling up empty time. Transition from γ^{4+} to $\tilde{\gamma}^{4+}$ is a spatial contour preserving deformation. Transition $\tilde{\gamma}^{4+}$ to γ^5 is a third change of meter $3/8 \longmapsto 6/8$. Finally, we will then use a Z transformation of $\gamma^{\tilde{5}}$ to modify some group of three quavers into a pointed crochet (γ'^5). In the new sequence we have a little *cantabile* melody. The transformations applied are an Hegelian-gestural equivalent of variational and developing strategies used by composers. In this way we have just completed a little musical clockwork, *The Hegel Song* as displayed in Figure 10.

7. Still More Symmetries? Future Developments

Reviewing the Hegel group $\mathcal{H}$, one might be tempted to extend it to the full symmetry group of the Hegel body $\mathcal{B}$, i.e., the automorphism group of this octahedron. There are two types of such automorphisms: (1) Geometric movements, elements of the special orthogonal group $SO(3, \mathbb{R})$ of 3D space, such as the 120^0 rotation around the axis through the centers of the triangles R, G, S and Z, F, N, see Figure 11. (2) Automorphisms in the orthogonal group $O(3, \mathbb{R})$ with determinant -1, such as the inversion $-Id = (GF)(RZ)(SN)$ which exchanges all our conceptual pairs. Why should one reject the first-case auto-morphisms that are not in $\mathcal{H}$? One reason could be that they have no fix-points, or, in other words: they are not bosonic actions generated by a selection of fermions, such as $R@Z$. A reason for avoiding automorphisms with determinant -1 (the case 2 above) could be that a change of orientation of the Hegel body $\mathcal{B}$ could be forbidden because human conceptualization is fundamentally using orientation, whatever that could mean in this embryonal state of mind.
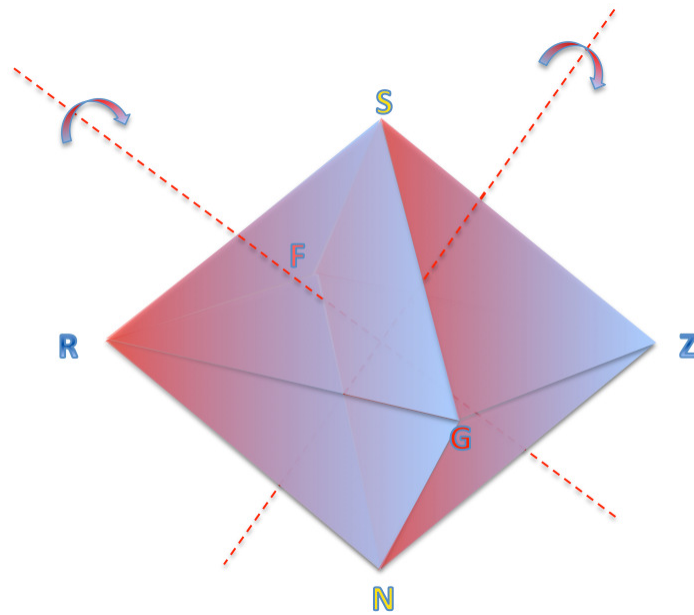
Fig. 11: Additional symmetries of the Hegel body might be considered, but at present, no philosophical interpretation is given.

We are not aware of any philosophical interpretation of additional symmetries, but the intrinsic geometry of $\mathcal{B}$ might create new aspects that could not have been conceived without this geometric rendering. Together with the problem of proving Escher-type theorems for (*SN*) and (*RZ*), this is a subject of future investigations.