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Symmetrical Balance and Coherence in Iranian Music and Visual Arts

A Comparative Study

> L'univers est construit sur un plan de la symétrie profonde de ce qui est en quelque sorte présent dans la structure interne de notre intellect.
>
> Paul Valéry[1]

Introduction

Everyone has some ideas about what symmetry is. We recognize the bilateral (left-right) symmetry of human body, of many other animals' bodies, and of numerous objects in our environment. We enjoy the rotation symmetry of many kinds of flower. Symmetry requires a reference frame, which is necessarily asymmetric. The absence of a reference frame implies identity, hence no possibility of change, and hence the inapplicability of the concept of symmetry.

Symmetry is a language by which many aspects of science and the arts can be expressed, appreciated, and eventually better understood and explained. Symmetry, from a denotative viewpoint, refers mainly to the equivalence in size or shape over a median plane or point. It essentially encompasses the idea of sameness and proportionality in reference to geometry. In fact, in line with this definition, a majority of individuals may characterize symmetry in mathematical or visual terms. When people hear the word symmetry, the first idea that enters their mind may be the symmetry commonly seen in geometrical figures found across mathematics or the harmonious proportionalities found in many of the great artworks throughout history. Symmetry, however, can exist in a much more abstract state, one that requires a stronger eye towards interpretation and contemplation. Such symmetry

[1] «The universe is built on a plan the profound symmetry of which is somehow present in the inner structure of our intellect», cited from http://www.phrases-anglais.net/aphorismes/paul-valery.

can exist in places people easily overlook: patterns of equivalence can be found in anything, from social interactions to music to literature.[2]

Symmetry is defined in two different ways: first, as a regular arrangement of equivalent elements,[3] and second, as the invariance of one object under the action of transformations.[4] Based on the first definition, symmetry is the result of the fact that each of the equivalent elements is surrounded by other equivalent elements in the same manner. The second definition – which can be classified as deductive – describes the symmetry of an object as its resistance (invariance) with regard to a certain change (non-identical transformation, i.e., a transformation that does not copy each point in itself).[5]

Why do we need to study the symmetry in music? A symmetry analysis of a musical piece will help us to describe the various aspects of symmetry which occur at different organizational levels, ranging from rhythmic repetitiveness and melody, counterpoint, and harmony symmetry, to symmetry of form. As one of the basic laws of nature, symmetry is integrated into all materials, intellectual and emotional creations of the human mind. The same is true of all art forms. In the philosophy of the Indo-Iranian people regularity and symmetry are the basis of "good form".

The growing interest in music symmetry was influenced by the book *Symmetry* of 1952 by Nobel-prize winning author Hermann Weyl.[6] Following this work, other scholars wrote on this issue, among them Lev Abramovič Mazel' and Viktor Abramovič Cukkerman in 1967,[7] Aleksej Vasil'evič Šubnikov and Vladimir Aleksandrovič Kopcik in 1972,[8] Guerino

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[2] Cf. Francis Antony, *Symmetry and Literature*, «Aspects of Symmetry», V (2010), pp. 5-9, http://www.uic.edu/honors/learning/AspectsV.pdf.

[3] Cf. Marjorie Senechal, *Symmetry Revisited*, in *Symmetry 2. Unifying Human Understanding* (part 1), edited by István Hargittai, «Computers and Mathematics with Applications», XVII, 1-3 (1989), pp. 1-12.

[4] Cf. Joe Rosen, *Symmetry Rules. How Science and Nature Are Founded on Symmetry*, Berlin – New York, Springer, 2008.

[5] Cf. Jadranka Hofman-Jablan, *Simetrija muzičkog dela*, Beograd, Zadužbina Andrejević, 1996; Eng. trans. by Ana Cuturilo Smjlianić, *Symmetry of Musical Work*, 2006, § "Definitions of symmetry", http://www.mi.sanu.ac.rs/vismath/jadrbookhtml.

[6] Hermann Weyl, *Symmetry*, Princeton, Princeton University Press, 1952.

[7] Lev Abramovič Mazel' - Viktor Abramovič Cukkerman, *Analiz muzykal'nych proizvedenij* ("Analysis of music compositions"), Moskva, Muzyka, 1967.

[8] Aleksej Vasil'evič Šubnikov - Vladimir Aleksandrovič Kopcik, *Simmetrija v nauke i iskusstve*, Moskva, Nauka, 1972; Eng. trans. *Symmetry in Science and Art*, New York – London, Plenum Press, 1974.

Mazzola in 1990,[9] Mária Apagyi in 1989,[10] Pozzi Escot the same year,[11] Siglind Bruhn[12] or Katalin Fittler in 1992.[13]

Types of Balance

There are numerous ways to categorise how elements of a piece are balanced. I suggest three different ways of balancing the constructing elements in any art works. The three types include: symmetrical equivalence, asymmetrical equivalence, and asymmetrical inequivalence. Symmetrical equivalence, or perfect symmetry, is mostly seen in Oriental visual designs and is quite rare in drama and music. Asymmetrical equivalence is a type of balance in which one element is balanced by another element having the same weight but a different shape. This type of balance is very useful in music and drama. The third type, asymmetrical inequivalence, occurs when a small element with extraordinary importance can bring balance to a rather larger element with a lower rank of importance. Diagrams in Figure 1 illustrate the three types of balance:

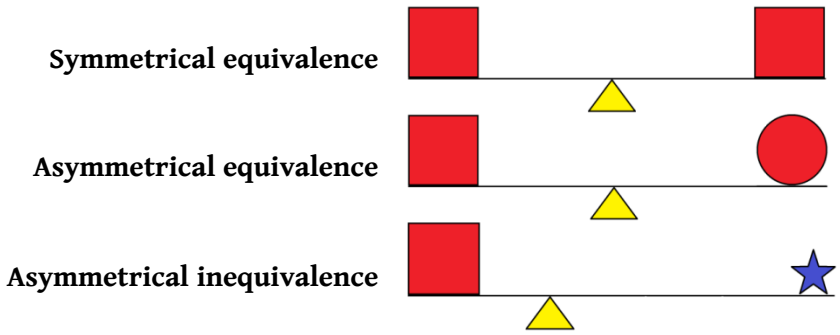


Fig. 1: Types of balance.

[9] Guerino Mazzola [et al.], *A Symmetry-Oriented Mathematical Model of Classical Counterpoint and Related Neurophysiological Investigations by Depth EEG*, in *Symmetry 2. Unifying Human Understanding* (part 2), edited by István Hargittai, «Computers and Mathematics with Applications», XVII, 4-6 (1989), pp. 539-594.

[10] Mária Apagyi, *Symmetries in Music Teaching*, *ivi*, pp. 671-695.

[11] Pozzi Escot, *The Symmetry of Music*, in *Symmetry of Structure*, abstracts of the interdisciplinary symposium (Budapest, 13-19 August 1989), edited by György Darvas and Dénes Nagy, Budapest, International Society for Interdisciplinary Studies of Symmetry, 1989.

[12] Siglind Bruhn, *Symmetry and Irreversibility in the Musical Language(s) of the Twentieth Century*, in *Symmetry of Patterns*, proceedings of the Hiroshima Symposium (part 2), «Symmetry», III, 2 (1992), pp. 187-201.

[13] Katalin Fittler, *Sator Arepo Tenet Opera Rotas, or Symmetrical Structures in Webern's* Reihen, «Symmetry», III, 3 (1992), pp. 253-263.

Isometric transformations and symmetry groups

The first steps in the development of the theory of symmetry in 18th century stem from basic isometric transformations (or mirror reflections, see Figure 2). Their combinations create all kinds of plane isometric transformations (transformations of congruence, or simply isometries). A geometric transformation is an isometry (transformation of congruence) if it preserves the distance (metrics), i.e., if each pair of points X and Y are transformed into points X1 and Y1 in such a way that the distance between points X and Y is congruent to the distance between X1 and Y1.[14]

Fig. 2: A symmetric object (dome of Sheikh Lotfollah Mosque in Esfahan, 1602) which consists of congruent parts (1-32 in first level, 1-16 in second level, and 1-8 in third level) arranged in the same manner. (Source: I*rania. Documentary and Pictorial Source of Information, Education and Entertainment About the Iranian People, Historical Sites and Vistas*, Tehran, Tehran University, multimedia cd, 1997).

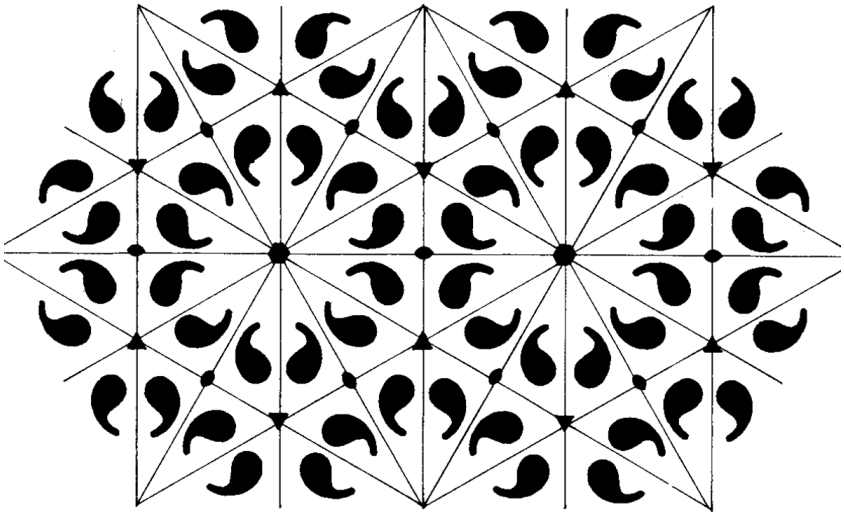
[14] Cf. Dragomir Lopandić, *Geometrija. Za III razred usmerenog obrazovanja matematičko-tehničke struke* ("Geometry. The Third Grade of Secondary Education Mathematics and Engineering Profession"), Beograd, Naučna knjiga, 1988; George Edward Martin, *Transformation Geometry. An Introduction to Symmetry*, Berlin – New York, Springer, 1982.

Symmetry in the Arts

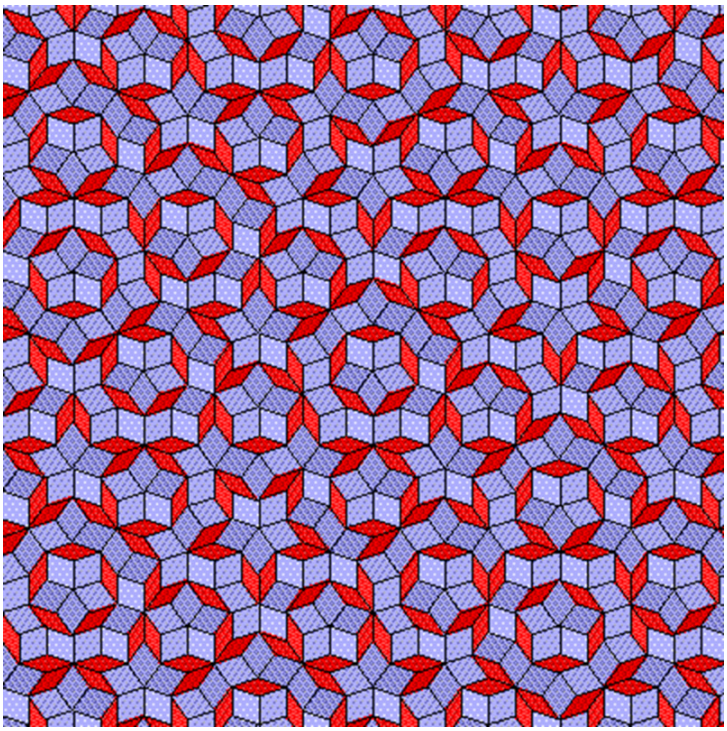
While the study of symmetry in the arts in Eastern countries has a long history, an explicit study of the role of symmetry in Western arts begins only with the turn of the 20th century. The reason for this can be found in the fact that classical aesthetics has always been more preoccupied with the interpretation of musical content than with its exact formal analysis, a result of the study of aesthetics in modern times which affirms an analytic approach in art. On the other hand, for an analysis of this kind to be carried through, a necessary prerequisite is the existence of theoretical assumptions and methods, which, in the case of the theory of symmetry, were formulated in 19th and 20th centuries. In its implicit form, symmetry is present in all works of art, and it represents a dominant component of all aesthetic organizational principles and rules of form.

At first sight, the cited definitions of symmetry may seem sufficiently wide, but they are applicable only to a limited number of works of art. Isometric groups of the symmetry of rosettes, friezes and ornaments apply only to perfectly regular isometric periodic structures such as crystals or ornaments (plane patterns). Therefore they have a very limited application in the analysis of structural laws in art. Thus the first and practically the only art discipline, which permits symmetry analysis from the aspect of isometric groups of symmetry is ornamental visual art. In other arts it is very rare that the same construction rule has been applied throughout the entire work with a uniform repetitive structure as a result. Although one may find examples of such structures in ornamental art, architecture or rhythmic patterns in music, it is much more frequent to come across exceptions to the rules of crystallographic symmetry, resulting in local, as opposed to global symmetries.[15] The global symmetry of a work of art is present in its overall formal elements, which unite all the structural levels of the work. At the other structural levels local symmetries are usually dominant (compare the two images in Figure 3: 3a, global symmetry in Persian wood art; 3b, local symmetry in a geometric design).

[15] Cf. J. Hofman-Jablan, *Symmetry of Musical Work*, cit., § "Specific properties of symmetries in arts and generalizations of the theory of symmetry".



(a)



(b)

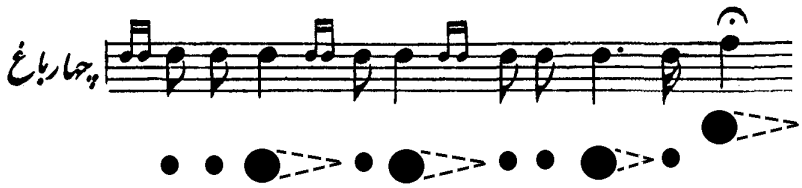
Fig. 3: The difference between global symmetry (a) and local symmetry (b). In (b), in spite of the existence of local symmetric subparts and their repetition, there is no common (global) rule governing all elements of the structure, which hence results to be aperiodic. This is not the case with image (a). (Source: Author's personal collection.)

An example of local symmetry in music is inversion or retrograde inversion, which offers a symmetric subpart, or the repetition of a theme in different parts of the musical piece.

One must, however, bear in mind that even in nature there is no perfect symmetry: for instance, the two sides of human face are not perfectly symmetric. It is possible in art to speak about approximate or relative symmetry. In this case equivalent elements are those, which are sufficiently common or close to one another, which is why we experience them as equivalent. Thus, when we approach the symmetry analysis of a work of art, we must choose our level of abstraction beforehand, i.e., we must determine the permitted limits of deviation from perfect regularity.

Symmetry in Musical Elements and Visual Arts

Symmetry in music can be studied from two perspectives: 1) visual modelling of musical works and their symmetry structure; 2) auditory symmetry in musical elements. When conducting a symmetry analysis of different structures it is almost always necessary to recur to their visual modelling. Of course, doing an auditory analysis demands capturing the ongoing sound of music in frame of time and drawing the symmetrical connection among its elements. It is not impossible to do it, but one cannot write down such experience. Visual models connect a musical piece and its visual diagram interpretation, which allows the study of its symmetry structure. See Example 1 from *āvāz Abū'atā* and compare it with Figure 4, showing Sheikh Lotfollah Mosque (1602).



Ex. 1: Visual transcription of first line of Chahārbāgh in āvāz Abū'atā from Radīf of Ab al-Hassan Sabā.[16]

Fig. 4: Sheikh Lotfollah Mosque (1602), Isfahan. (Source: *Irania*, cit.)

[16] Ab al-Hassan Sabā, *Dureh-i duvvum santūr* ("Level 2 for Santūr"), Tihrān, Montakhab Sabā, 1985, II: *Radīf Sabā*, p. 16.

Rhythm

Rhythm is the «organization of time in parts accessible to the senses».[17] Most natural laws and occurrences, such as the coming and going of waves, the change of day and night, the changing of seasons, breathing, heartbeat, pendulum movements, etc., are all different manifestations of periodicity in time.

Studied from a geometric point of view, rhythmic structure is one-dimensional (linear). If we take the motif as a basic rhythmic and melodic entity and we explore the changes of its rhythmic structure within the study of a motif, we can follow different symmetry transformations. According to Hofman-Jablan,[18] all symmetry transformations of the rhythmic structure can be classified into two basic types:

- absolute transformations: all elements of the rhythmic structure subjected to their action are transformed in the same way;
- relative transformations which act in different ways on different elements.

In the first case we can examine two kinds of transformations: isometric transformations (transformations of congruence) and proportional transformations (transformations of similarity).

Isometric transformations of rhythmic structure are translational repetition *t* (translation, or literal repetition), and retrograde repetition *m* (mirror reflection).

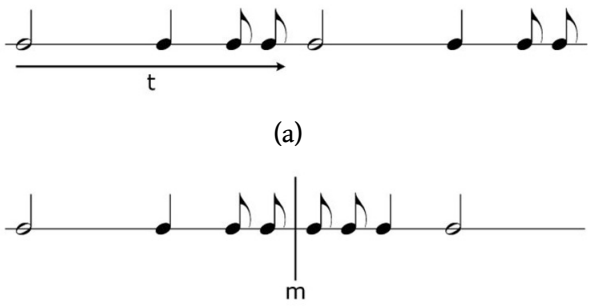


Fig. 5: a) Translational repetition (translation t);
b) retrograde repetition (mirror reflection m).[19]

[17] Andreas Heusler, *Deutsche Versgeschichte*, «Grundriss der germanischen Philologie», VIII (1925), p. 17.

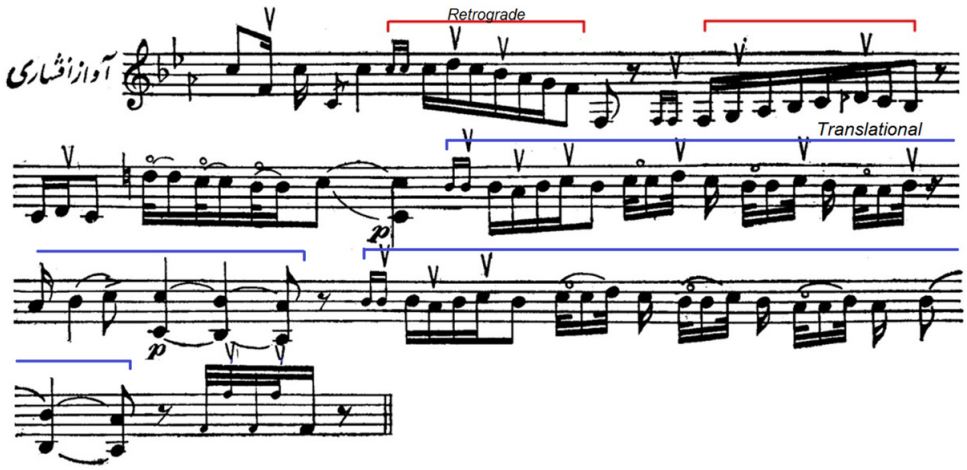
[18] J. Hofman-Jablan, *Symmetry of Musical Work*, cit., § "Absolute and relative transformations of rhythmic structure".

[19] *Ibidem*.

(a)

(b)

Fig. 6: a) Manaf'i Hayavan of Ibn Bakhtish': Two Gazelles. Maraghe, c. 1300; b) Two peacocks designed on Persian tiles at the front door of a mosque in Qajar period.



Ex. 2: Retrograde repetition (mirror reflection) *āvāz Afshārī* from *Radīf* by Abulhassan Sabā.[20]

[20] A. Sabā, *Dureh-i duvvum santūr*, cit., II, p. 17.

- U - -	(vs.)	- - U -
Fāelāton (long, short, long, long)		Mostafʿalon (long, long, short, long)

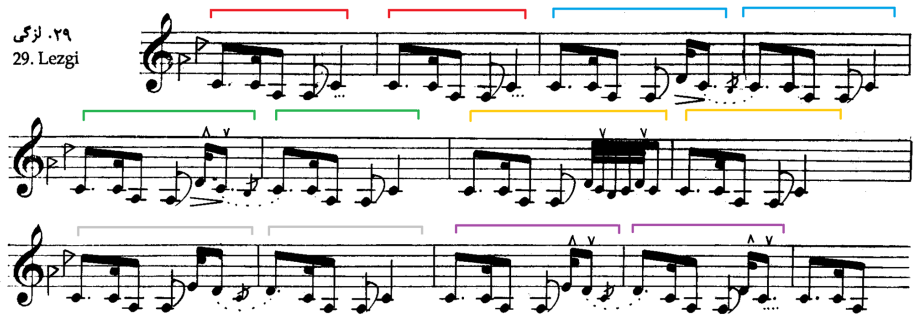
Fig. 7: Retrograde repetition (mirror reflection) in a Persian poetic motif.

One can draw a parallel of the idea of rhythmic symmetry in Persian poetic motives taken from the *ʾArūzi* system and that of motives of Iranian music rhythmic cycles (*Adwār īqāʿī*). The following example is a rhythmic cycles in *īqāʿī* system reported in a 13th-century manuscript by Qutb al-Dīn Shīrāzī (1236-1312) called *Durrat al-Tāj*. In these rhythmic cycles the first three motives in their repetition appear in a symmetric way. Figure 8 illustrates the way the motives of this cycle arrange symmetrically.

Atānīn	Tanan	Tanan	Tananan		Tananan	Tanan	Tanan
Pattern of shorts and longs	U -	U -	UU -		UU -	U -	U -
Symmetry of motives	←						→

Fig. 8: Rhythmic Cycle of *Turkī*, based on the version reported by Qutb al-Dīn Shīrāzī in *Durrat al-Tāj*.[21]

While the most popular repetitions in Persian visual arts are retrograde repetitions (mirror reflections), the most common repetitions in Persian music are translational repetitions – retrograde repetitions are very rare in Persian music.



Ex. 3: Translational repetition in *Lezgi in dastgāh chahargāh* from *Radīf* by Mirza ʿAbdullāh.[22]

[21] Mahmūd ibn Masʿūd Qutb al-Dīn Shīrāzī, *Durrat al-tāj li-ghurrat al-dibāj*, edited by Seyed Hasan Mishkat, Tihrān, Vizārat-i Farhang, 1945, II, p. 129.

[22] Mirza ʿAbdullāh, *Le répertorie-modèle de la musique iranienne. Radīf de Tar et de Setar de Mirza ʾAbdullah (version de Nur ʾAli Brumand)*, transcription by Jean During, Tihrān, Sorush, 1991, p. 202.

Permutational equality and permutational proportionality of rhythmic entities

The permutation of the elements of a set X (for instance, note durations long-long-short-short) is any sequence that can be formed from its elements. Here, we can get six permutations from it as follows:

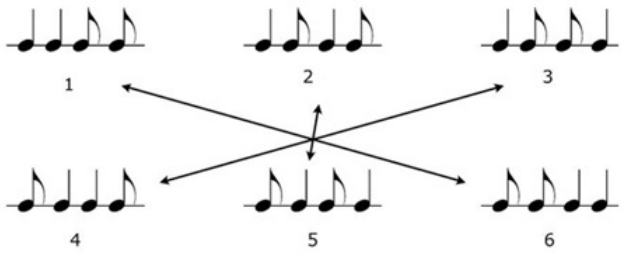
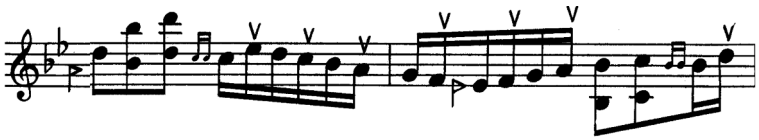


Fig. 9: Sequences of permutation long-long-short-short.[23]

Symmetry of music meter

A special element of the symmetry of rhythmic structure is the symmetry of the musical meter – the symmetry regularities, which appear as the order of accented and unaccented parts of the meter. In all these cases it is reduced to a linear structure within which repetition (translation) and retrograde repetition (mirror reflection) can appear.

The order of accented beats can also be studied from the aspect of antisymmetry (accented part-unaccented part).



Ex. 4: Metric symmetry in *chahārmezrāb Bayāt tūrk* by replacing the long notes from the accented beat of the first bar to unaccented beat of the second bar.[24]

[23] J. Hofman-Jablan, *Symmetry of Musical Work*, cit., § "Permutational equality and permutational proportionality of rhythmic entities".

[24] Farāmarz Pāyvar, *Siqet'eh Chahārmezrāb* ("Thirty Piece of Chahārmezrāb"), Isfahan, Vahed-i Sorud va Mūsiqī Farhang va Ershād Islamī, 1990, p. 27.

Fig. 10: Metric symmetry in Iranian architecture (Shiraz Jāme' Mosque).

The symmetry of melodic structure

The first issue in symmetry of melody is to study the order and quality of intervals. Three elements ought to be examined: the starting point, the value, and the orientation of the interval. Each interval is defined by the starting pitch and the difference in pitches between the last the starting note. If we consider the starting point of any melodic contour as the base level, for ascending intervals the starting note is positive and for descending intervals it is negative.

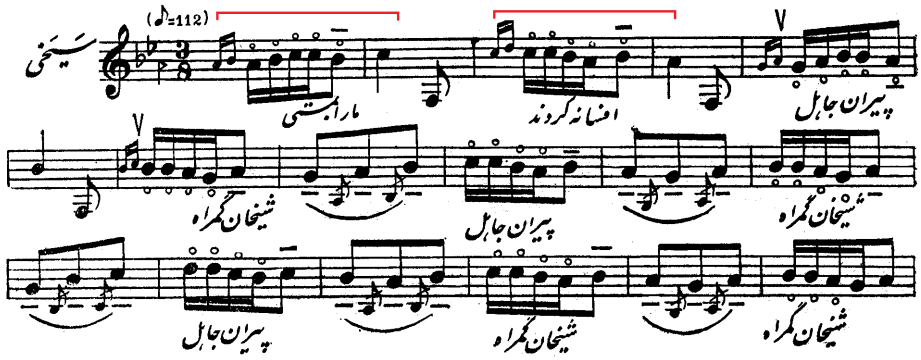
The symmetry transformations of a melodic structure can be divided into two basic types: absolute and relative. The concept of absolute transformations of a melodic entity implies that the transformations have the same impact on all the elements of the melodic entity and, consequently, on all interval relations occurring among them. Absolute transformations preserve the order of the elements. By contrast, in the case of relative (or approximate) symmetry transformations of a melodic structure, the structure of interval relationships among the notes breaks off.

There are four types of transformations of similarity (proportionality):

- proportional transposition;
- proportional retrograde transposition;
- proportional inversion;
- proportional retrograde inversion.

In the case of relative (approximate) symmetry transformations of a melodic structure, we may continue to look for invariants of transformations, i.e., for the occurrence of certain regularities within relative symmetry transformations. The solution to this problem is analogous to the solution of permutational equivalence of rhythmic structures. In nature, and especially in art, ideal symmetry occurs very rarely. This is why in music similarity symmetry, i.e., relative symmetry transformations prevail. For example, instead of absolute (strict) inversion, free inversion usually occurs.

In Iranian traditional music we rarely come across absolute symmetry while we can find the relative symmetrical repetitions in various ways in *gūshe-hā* of Persian *radīf*. See the approximate inverted repetition in *gūshe* of Sayakhi in Example 5.

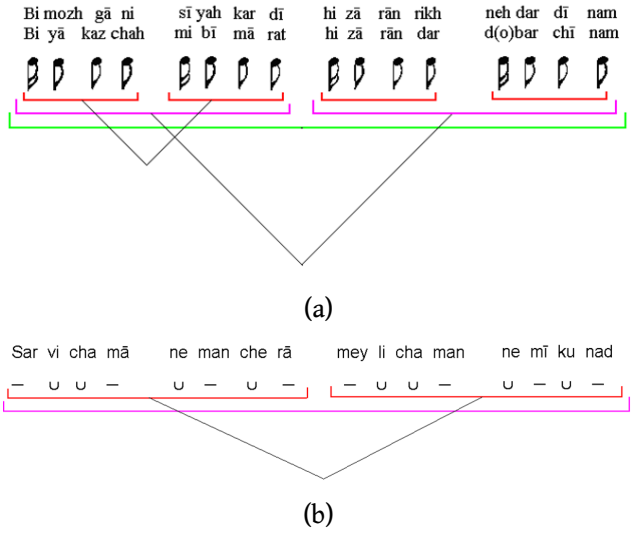


Ex. 5: The significance of a two bar motif in the *gūshe* of Sayakhi from *Radīf* of Sabā and its approximate inverted repetition response.[25]

[25] A. Sabā, *Dureh-i Duvvum Santūr*, cit., II, p. 11.

Phrasing Balance and Grouping

The formal structure of phrasing in Iranian music is very much in debt to Persian poetry. In Persian poetry balance comes to exist by symmetry of rhythmic motives and of every verse and line. Every phrase usually is made by combining two motives. The lines can be made of four motives. Motives can be single, double or quadruple. The first following example shows a poem, which is shaped by a single motif and the second example is a poem shaped by combining two different motives. The balances in both examples come from their symmetric repetition.



Ex. 6: Balances between poetic motives and phrases in Persian poems.

In Persian architecture, small motivic elements join together to build the greater segments of the form. Formal structure of Pol-e Khajū (Khajū Bridge, see Fig. 11) in Esfahan uses the same idea of combining motives to build greater segments while keeping symmetrical balance.

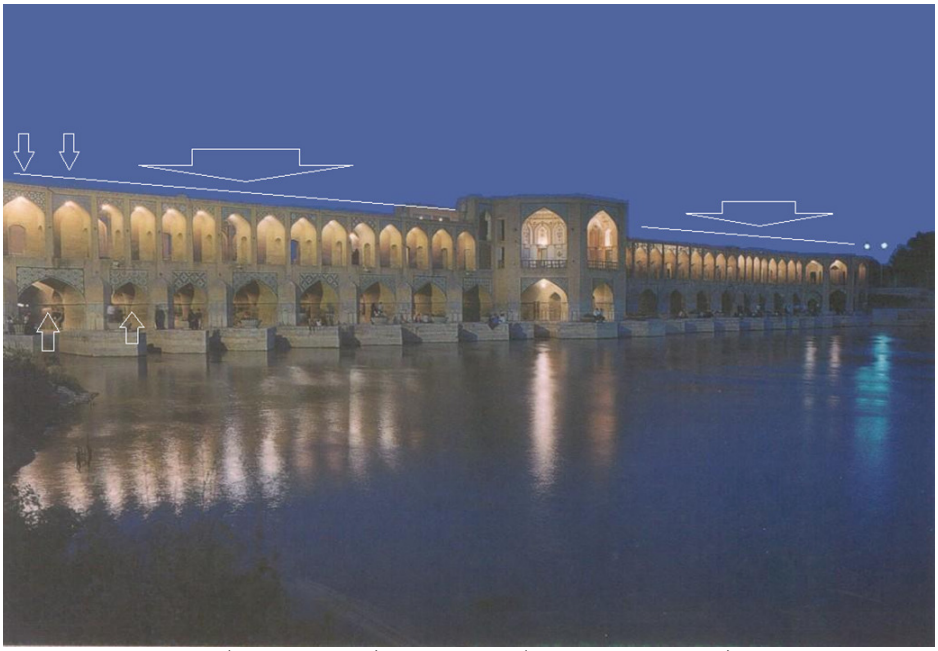


Fig. 11: Pol-e Khajū (Khajū Bridge) in Esfahan. (Source: *Irania*, cit.)

Conclusion

Symmetry is an important visual property for humans and for organisms in general. It is useful for discriminating living organisms from inanimate objects, for identifying face orientation and the direction of attention. Its importance to humans is expressed in its ubiquitous use as a design principle in objects of human construction (from buildings to Persian rugs).

In the philosophy of Indo-Iranian people, regularity and symmetry are the basis of "good form". It is hard to find a Persian traditional building through history of Iranian architecture, which has not totally or partially used symmetrical arrangements among its formal elements.

The combinations of isometric transformations (mirror reflections) create all kinds of plane isometric transformations. Geometric transformation is an isometry (transformation of congruence), which is very popular in Persian architecture. Yet, the first and practically the only art discipline which permits symmetry analysis from the aspect of isometric groups of symmetry is ornamental visual art. In other arts it is very rare for the same rule of construction to be applied throughout the entire work of art with a uniform repetitive structure as a result.

It is possible in art to speak about approximate or relative symmetry. In this case equivalent elements are those which are sufficiently common or close to one another, which is why we experience them as equivalent. Thus, we can conclude that all symmetry transformations of the formal structure can be classified into two basic types: absolute and relative transformations. Relative transformation may take place in numerous ways depending on the creativity of the artist. The absolute transformation comes to exist in two different ways: translational repetition (translation, or literal repetition) and retrograde repetition (mirror reflection).

The formal structure of Persian poetry has had a great influence on shaping Iranian music. The traditional genres of Persian poetry use symmetric motives as the building blocks for their construction. These blocks are called *arkan-e 'arūzī* and provide several rhythmic motives with various combination of short and long durations. Rhythmic structure of Iranian music abundantly borrows symmetrical patterns of rhythmic motives from the form of Persian poetry. As suggested in this paper, the way of using such motives is very similar to the use of visual motives and geometric designs in Persian visual arts. This similarity generates this fact that symmetry plays more or less the same role in various aspects of Persian artistic culture: poetry, music and visual arts.

Note

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