Luigi Verdi

Kaleidocycles and Rhythmic Canons

Introduction

Among the main purposes of the theory of musical kaleidocycles is the setting and organization of chord groups on the basis of prearranged rules: the fundamental theoretical principle is the transformation of chords following a transposition cycle. It is possible to generate links according to ordering rules such as, for instance, common notes. However, kaleidocycles may also be used as a "combinatorial" tool to build larger sets, for example the chromatic aggregate. The complementarity relation towards a reference set is a key principle of the kaleidocyclical system.[1]

Starting with the analysis of traditional temperament, the theory of musical kaleidocycles elucidates the fundamental numerical rules governing a chord structure, which may have compositional implications, such as when processing kaleidocycles into rhythmic-melodic canons. From a formal point of view, the composition technique of the rhythmic canons may directly be derived from a kaleidocyclical pattern. It is possible to put one interval structure together with a periodical rhythm which will have a chosen minimal unit as its fundamental beat and a submultiple of the sum of the items of the corresponding interval structure as its period. This recalls other compositional strategies relating to the kaleidocyclical technique, sharing with it the periodical structure and the strict numerical approach; thus you can find links with some apparently heterogeneous composition

1 For an introduction to the concept of a kaleidocycle with definitions, see: Luigi Verdi, *Musical Kaleidocycles: Composition and Analytical Techniques*, «Analitica. Rivista Online di Studi Musicali», IV (2007), http://www.gatm.it/analitica/numeri/volume4/numerounico/0en_1.htm accessed on 3 June 2016. Other references: Moreno Andreatta, *Méthodes algébriques dans la musique et musicologie du XXème siècle: aspects théoriques, analytiques et compositionnels*, thesis in computational musicology, Paris, Ehess/Ircam, 2003; Id., *Calcul algébrique et calcul catégoriel en musique: aspects théoriques et informatiques*, in *Le calcul de la musique*, ed. by Laurent Pottier, Saint-Étienne, Publications de l'Université de Saint-Étienne, 2008, pp. 429-477; Id. – Dan T. Vuza, *On Some Properties of Periodic Sequence in Anatol Vieru's Modal Theory*, «Tatra Mountains Mathematical Publications», XXIII (2001), pp. 1-15; Guerino Mazzola, *The Topos of Music: Geometric Logic of Concepts, Theory, and Performance*, Basel, Birkhauser, 2002; Robert Morris, *Composition With Pitch-Classes: A Theory of Compositional Design*, New Haven, Yale University Press, 1987; Luigi Verdi, *Organizzazione delle altezze nello spazio temperato*, Treviso, Diastema – Ensemble 900, 1998; Id., *Caleidocicli musicali. Simmetrie infrante dei suoni*, new edition, with essays by Moreno Andreatta, Carmine Emanuele Cella, Renzo Cresti, Giovanni Guanti, Milano, Rugginenti, 2010; Dan T. Vuza, *Supplementary Sets and Regular Complementary Unending Canons,* «Perspectives of New Music», XXIX, 2 (1991), pp. 22-49; XXX, 1 (1992), pp. 184-207; XXX, 2 (1992), pp. 102-124; XXXI, 1 (1993), pp. 270-305.

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systems such as, for example, Anatol Vieru's modal theory, Iannis Xenakis's sieve theory, as well as the atonal composition technique developed by the American theorist and composer George Perle. In all of the three situations, the concept of cycle is the foundation of a composition theory, and the numerical translation of some events leads to results which can be differently applied both in the vertical space in terms of pitch arrangement and in the horizontal as regards the rhythmic-melodic patterns.

A strict algebraic formalization of Anatol Vieru's modal theory was proposed by the Romanian mathematician Dan Tudor Vuza, who was able to define the canons of maximal category as canons where all voices are complementary in the sense that there are neither intersections nor gaps between them: taking one of these canons as the starting point, it is possible to demonstrate that similar canons can be built with kaleidocycles.

As an example, we can build a canon of Vuza's according to the kaleidocyclical system in a tempered space where the octave is divided into 72 parts (twelfths of a tone).

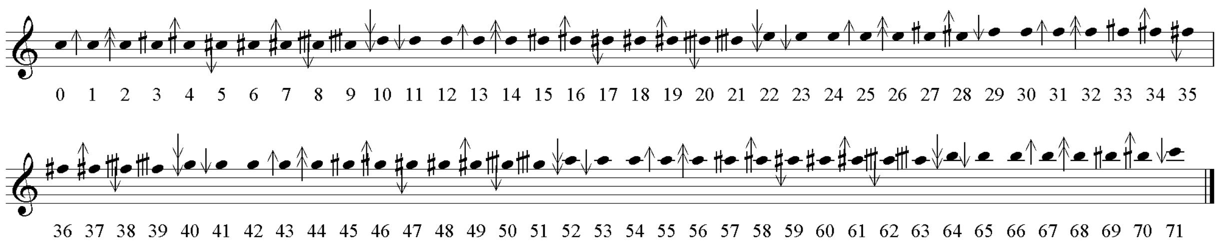

Fig. 1: Tempered space where the octave is divided into 72 parts (twelfths of a tone)

Numerical Vector of Common Notes

The vector can be identified by adopting some reference values, as follows.

A set is chosen and the intervals separating the notes determined (a twelfth of tone = 1), beginning from a given starting point "C" as identified by arranging pitches with the shortest intervals on the left:

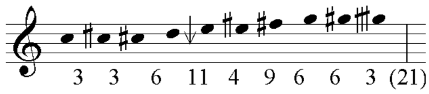

Fig. 2: Twelfths of a tone separating the notes of a chosen set

If a reference pitch of the set is given (C=0), a series of numbers indicates the notes, where a twelfth of a tone equals 1.

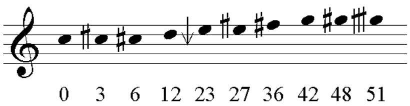

Fig. 3: Series of numbers indicating all notes of the previous set

Thus, the set is (0,3,6,12,23,27,36,42,48,51) and its interval structure, i.e. the series of twelfths of a tone separating every note, is (3-3-6-11-4-9-6-6-3-(21)), where the last number in brackets indicates the interval required to complete the octave. The sum of digits of the interval structure must be equal to 72.

Now it is necessary to identify the "numerical vector of common notes" of the given set. The vector (number of notes which are shared by the various transpositions) can be calculated by means of a Cartesian axis reporting all 72 transpositions arranged one upon the other: on the abscissa are lined the pitches, on the ordinate the common notes upon 72 transposition levels. The arrangement of common notes upon various transposition levels is presented in the following transposition diagram, where every diagonal line represents a single note transposing on various levels: since a diagonal intersects a vertical, representing a set note on 0 level, a common note with the original and its transposition is obtained. In the right column are indicated the numbers of common notes between the set and all its transpositions: this is the numerical vector of common notes.

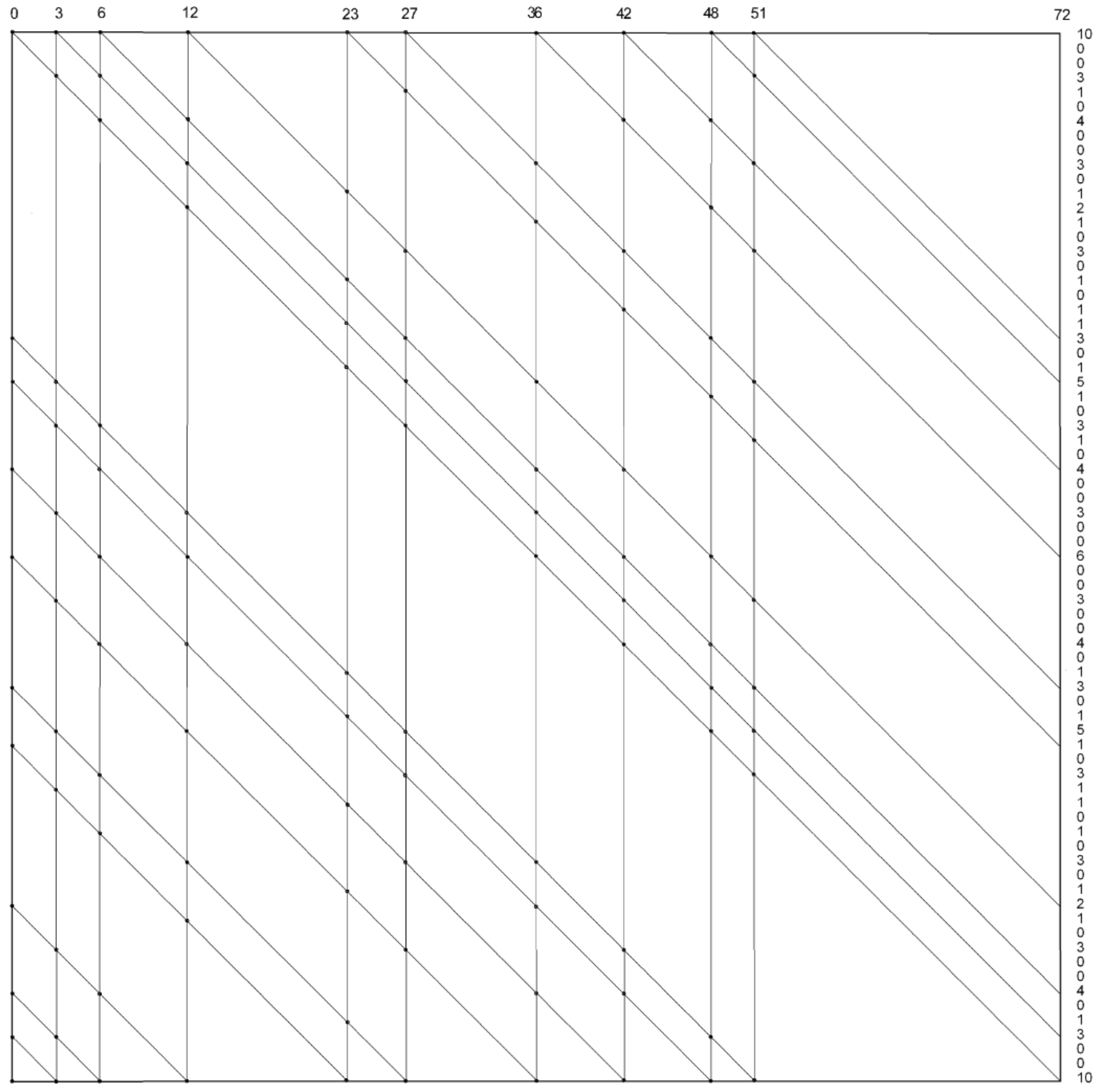

Fig. 4: Transposition diagram of (0,3,6,12,23,27,36,42,48,51) common notes vector

The vector can be also represented by a transposition chart:

transposition	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
numerical vector of common notes	10	0	0	3	1	0	4	0	0	3	0	1	2	1	0	3	0	1	0	1	1	3	0	1

transposition	24	25	26	27	28	29	30	31	32	33	34	35	**36**	37	38	39	40	41	42	43	44	45	46	47	48
numerical vector of common notes	5	1	0	3	1	0	4	0	0	3	0	0	**6**	0	0	3	0	0	4	0	1	3	0	1	5

transposition	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
numerical vector of common notes	1	0	3	1	1	0	1	0	3	0	1	2	1	0	3	0	0	4	0	1	3	0	0	10

Fig. 5: Transposition chart of (0,3,6,12,23,27,36,42,48,51) common notes vector

Interval Cycle

A link consisting of groups of periodically repeating intervals originates a *cycle*. The group of repeating intervals is the *module* of the cycle.

The module repeats itself upon a unit interval named *base*, given by the sum of the elements of the module; in the case of a sample module (14-8-10-8-14-18), the base amounts to 72: since a base can assume only values from 0 to 71, other values may be reduced to an octave by one operation named mod(72). In the previous case the base is equivalent to 0 (since 72=0), and the subsequent module repetition will be set on the interval 0. The number of intervals forming a module is labelled *meter*; in the previous case, a cycle upon module (14-8-10-8-14-18) is metrically structured 6. The unit of all necessary repetitions for a module to close its cycle is labelled *period*: the period number is calculated by adding the base to itself, until forming a 72-multiple; for example, module (14-8-10-8-14-18) placed upon base 0 appears only once before going back to the initial level, giving rise to a period-1 cycle, since 14+8+10+8+14+18 = 72.

Every group of intervals can be graphed as a set of segments inscribed in a regular 72-sided polygone. A segment is the bidimensional projection of an interval. In graphic representation, the previous module consists of six segments.

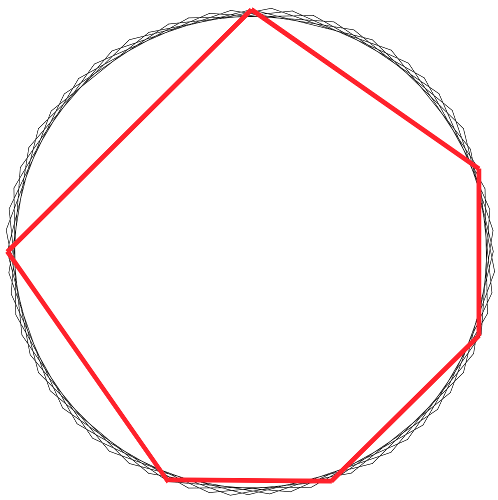

Fig. 6: Graphic representation of the (14-8-10-8-14-18) module

The previous module may also be represented in chart form:

module	14											
			8								10	
									8			
					14							
							18					

Fig. 7: Transposition chart of the 14-8-10-8-14-18 module

The various pitches upon which the base repeats itself in order to complete a period give rise to a *base class*. The base class is the sequence of levels necessary to complete its inner articulation until the completion of the period; starting with a 0 level, the module transposes itself on the base class, developing the *base module*. In the present case of period 1, base and base class are equivalent.

module	14-8-10-8-14-18
common notes	0-0-0-0-0-0
base	0

base module	(0,14,22,32,40,54)
base class	(0)

Fig. 8: Transposition chart of the 14-8-10-8-14-18 module into (0,14,22,32,40,54) base module

Application of a Chord to a Module

If the (0,3,6,12,23,27,36,42,48,51) set is applied to the chosen module 14-8-10-8-14-18, the set can be graphed as follows:

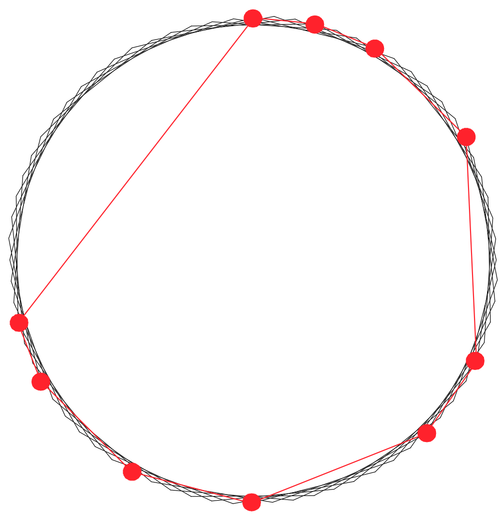

Fig. 9: Graphic representation of the (0,3,6,12,23,27,36,42,48,51) set

For the practical realization of the application, it is useful to develop a graphic scheme, by transcribing every transposed set vertically, according to a horizontal axis representing the base module. In the following chart, spots represent single notes and base module items are marked.

transposition levels (vertical)							
71							
70			o				
69							
68				o			
67					o		
66						o	
65		o					
64			o				
63					o		
62		o					
61							
60						o	
59				o			
58			o				
57						o	
56		o					
55				o			
54						o	
53							
52					o		
51	o						o
50		o					
49			o				
48	o						o
47							
46					o		
45			o				
44				o			
43					o		
42	o						o
41		o					
40					o		
39							
38				o			
37		o					
36	o						o
35				o			
34			o				
33						o	
32				o			
31							
30						o	
29							
28			o				
27	o						o
26		o					
25			o				
24						o	
23	o						o
22			o				
21							
20		o					
19					o		
18						o	
17		o					
16					o		
15							
14		o					
13							
12	o						o
11							
10					o		
9						o	
8				o			
7							
6	o						o
5						o	
4					o		
3	o						o
2				o			
1			o				
0	o						o
base module (horizontal)	0	14	22	32	40	54	0

Fig. 10: Chart of the (0,3,6,12,23,27,36,42,48,51) set applied to a 14-8-10-8-14-18 module

Every item of the base module gives rise to a canon entry upon the various transposition levels. Thus a *cyclical rhythmical scheme* is generated by the vertical development of the set upon its base, placed on the horizontal axis of the meter. The application of a chord gave rise to a new original structure named *kaleidocycle*, after some of Maurits Escher's graphic techniques. The kaleidocycle is the effect of a transformation of space into time, that is a vertical structure which changes into a horizontal one.

Tiling-Kaleidocycle (Vuza's Regular Complementary Rhythmic Canon)

On changing the vertical arrangement of the previous chart into a rhythmic and horizontal one, it is possible better to observe that a regular six-voice rhythmic canon is generated, where all voices are complementary without intersections among them, i.e. there is never any voice entry at the same time, because there are no common notes between (0,3,6,12,23,27,36,42,48,51) and its transpositions upon the 14-8-10-8-14-18 module:

transposition levels	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	base module
						o				o									o						o						o			o			54
					o						o						o			o																	40
			o						o			o																					o			o	32
		o																					o			o			o						o		22
															o			o			o						o										14
	o			o			o						o											o				o									0
transposition levels	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	base module

transposition levels	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	base module
																			o			o			o						o						54
					o			o			o						o											o				o					40
			o						o											o				o									o				32
										o				o									o						o						o		22
		o				o									o						o						o			o							14
	o						o						o			o																					0
transposition levels	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	base module

Fig. 11: Horizontal setting of the (0,3,6,12,23,27,36,42,48,51) chord applied to a 14-8-10-8-14-18 module. This has generated a six-voice complementary rhythmic canon (tiling-kaleidocycle)

Thus the previous vertical setting has given rise to a rhythmic and horizontal development, which corresponds to the following canon:

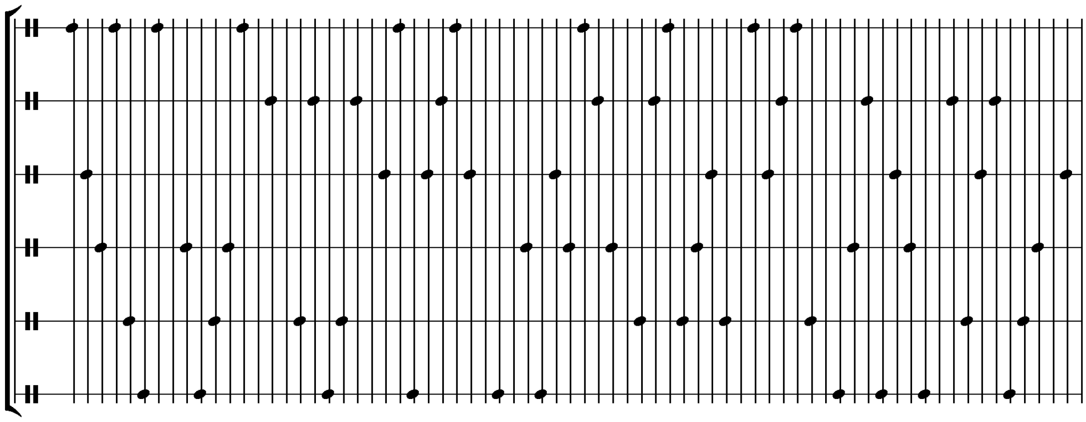

Fig. 12: Six-voice complementary rhythmic canon (tiling-kaleidocycle), generated by the application of a (0,3,6,12,23,27,36,42,48,51) chord to a 14-8-10-8-14-18 module.

Es. audio 1

Note that this is not a maximal category canon, since the chosen set consists of 10 pitches and some gaps remain in the rhythmic development; gaps all together are 12 because 72–(10x6)=12.

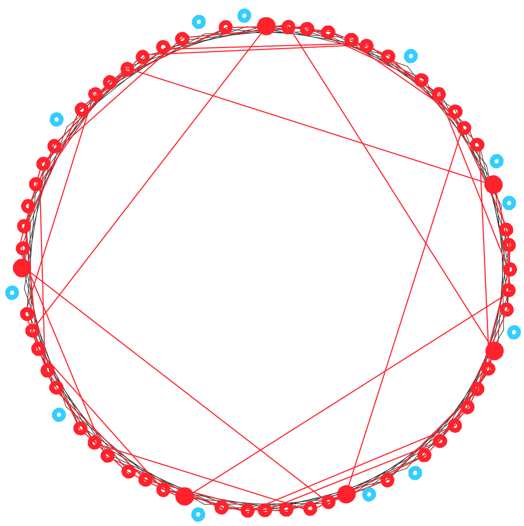

Fig. 13: Graphic representation of the previous tiling-kaleidocycle, with 12 gaps remaining

Fill-Kaleidocycle (Vuza's Maximal Category Canon)

To obtain a maximal category canon it is necessary to operate on a 12-pitch set, for example (0,1,5,6,12,25,29,36,42,48,49,53), whose interval structure, i.e. the twelfths of a tone separating the various notes, is 1-4-1-6-13-4-7-6-6-1-4-(19).

Fig. 14: Series of numbers indicating the (0,1,5,6,12,25,29,36,42,48,49,53) set

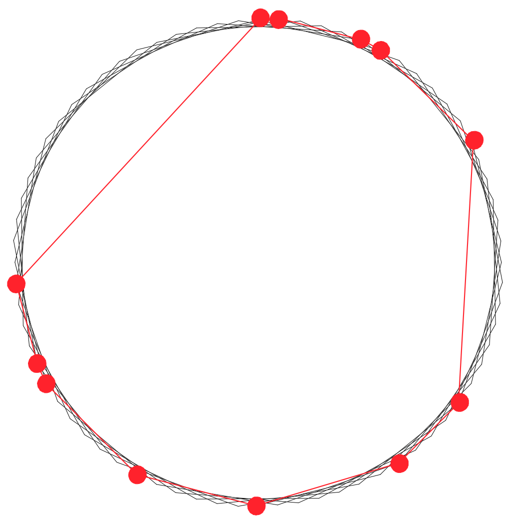

Fig. 15: Graphic representation of the (0,1,5,6,12,25,29,36,42,48,49,53) set

The resulting transposition diagram is as follows:

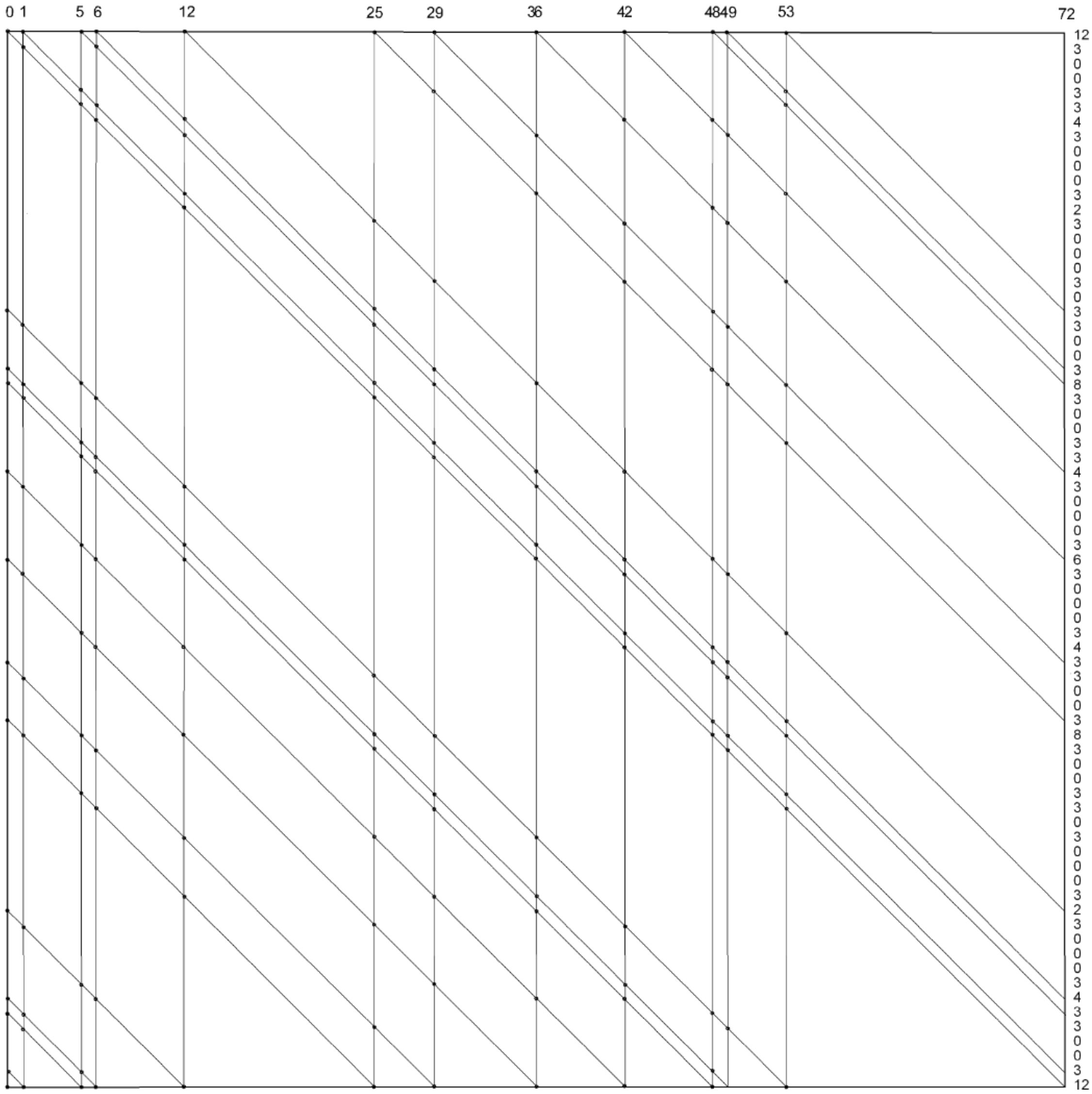

Fig. 16: Transposition diagram of (0,1,5,6,12,25,29,36,42,48,49,53) common notes vector

The vector can be also represented by a transposition chart:

transposition	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
numerical vector of common notes	12	3	0	0	3	3	4	3	0	0	0	3	2	3	0	0	0	3	0	3	3	0	0	3

transposition	24	25	26	27	28	29	30	31	32	33	34	35	**36**	37	38	39	40	41	42	43	44	45	46	47	48
numerical vector of common notes	8	3	0	0	3	3	4	3	0	0	0	3	**6**	3	0	0	0	3	4	3	3	0	0	3	8

transposition	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
numerical vector of common notes	3	0	0	3	3	0	3	0	0	0	3	2	3	0	0	0	3	4	3	3	0	0	3	12

Fig. 17: Transposition chart of (0,1,5,6,12,25,29,36,42,48,49,53) common notes vector

Now the (0,1,5,6,12,25,29,36,42,48,49,53) set is applied to module 14-8-10-8-14-18, which is built on transposition levels with 0 common notes, by transcribing every transposed chord vertically, according to a horizontal axis representing the base module (fig. 18).

On changing the vertical arrangement of the previous chart into a rhythmic and horizontal one (fig. 19), it is possible to observe that a regular six-voice complementary, rhythmic canon of maximal category arises, where all voices are complementary in the sense that there are neither intersections nor gaps between them. There are no intersections because there are no common notes between (0,1,5,6,12,25,29,36,42,48,49,53) and its transpositions upon the 14-8-10-8-14-18 module, and there are no gaps because the original set consisted of 12 notes, in such a way as to complete the 72 (12x6) total.

transposition levels (vertical)							
71			o				
70			o				
69					o		
68				o			
67		o					
66						o	
65					o		
64			o				
63		o					
62		o					
61				o			
60						o	
59						o	
58			o				
57				o			
56		o					
55						o	
54						o	
53	o						o
52					o		
51			o				
50		o					
49	o						o
48	o						o
47			o				
46					o		
45					o		
44				o			
43		o					
42	o						o
41					o		
40					o		
39		o					
38				o			
37				o			
36	o						o
35						o	
34			o				
33				o			
32				o			
31						o	
30						o	
29	o						o
28			o				
27			o				
26		o					
25	o						o
24						o	
23			o				
22			o				
21					o		
20		o					
19		o					
18						o	
17					o		
16					o		
15		o					
14		o					
13				o			
12	o						o
11						o	
10					o		
9				o			
8				o			
7						o	
6	o						o
5	o						o
4					o		
3			o				
2				o			
1	o						o
0	o						o
base module (horizontal)	0	14	22	32	40	54	0

Fig. 18: Chart of the (0,1,5,6,12,25,29,36,42,48,49,53) set applied to a 14-8-10-8-14-18 module

transposition levels	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	base module
								o				o							o						o						o	o				o	54
					o						o						o	o				o															40
			o						o	o				o																			o	o			32
				o																			o	o				o	o						o		22
															o	o				o	o						o										14
	o	o				o	o						o													o				o							0
transposition levels	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	base module

transposition levels	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	base module
																			o	o				o	o						o						54
					o	o				o	o						o													o				o			40
		o	o						o													o				o							o				32
												o				o							o						o						o	o	22
				o				o							o						o						o	o				o					14
	o						o						o	o				o																			0
transposition levels	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	base module

Fig. 19: Horizontal setting of the (0,1,5,6,12,25,29,36,42,48,49,53) chord applied to a 14-8-10-8-14-18 module. It gives rise to a six-voice complementary rhythmic canon (fill-kaleidocycle)

Thus the previous vertical setting gives rise to a rhythmic horizontal development, which corresponds to this Vuza canon of maximal category:

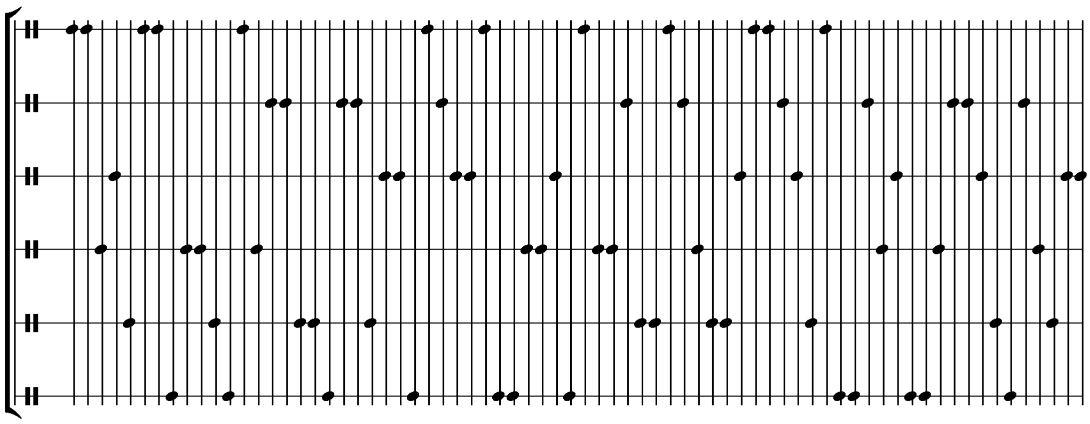

Fig. 20: Six-voice complementary rhythmic canon (fill-kaleidocycle), generated by the application of a (0,1,5,6,12,25,29,36,42,48,49,53) chord to a 14-8-10-8-14-18 module

Es. audio 2

It is a canon of maximal category, because no gaps remain between the voices:

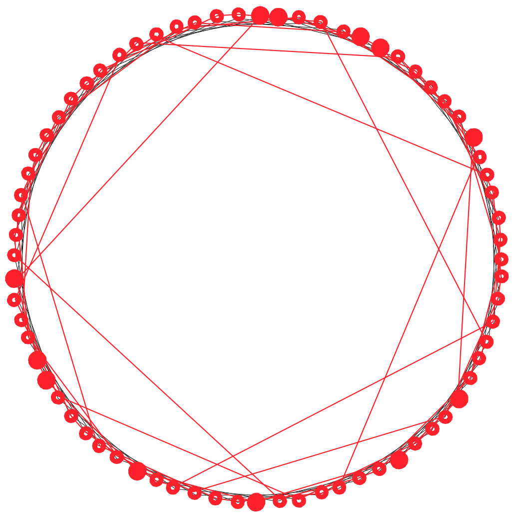

Fig. 21: Graphic representation of the previous fill-kaleidocycle; no gaps remain between the voices, because 12x6=72

NOTE