



La tradizione filosofica dall'antico al moderno

Rivista semestrale, Firenze-Parma, E-theca OnLineOpenAccess Edizioni

Anno IX, n. 3, 2022

**Logic and Modalities in the Late Middle Ages.
The Tradition of the Oxford Calculators**

A cura di Irene Binini

ISSN 2284-1180

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LOGIC AND MODALITIES IN THE LATE MIDDLE AGES. THE TRADITION OF THE OXFORD CALCULATORS

A CURA DI IRENE BININI

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ISSN 2284-1180

INTRODUCTION

IRENE BININI

The essays collected in this issue aim to explore several topics connected to the logical theories of the Oxford Calculators and the impact that they had on later developments in logic. They present new evidence and new interpretations on three main subjects that are central to the production of the Calculators, namely: (i) their theory of modalities and understanding of the nature of (im)possibility; (ii) their views on insolubles and semantic paradoxes; and finally (iii) their approach to epistemic sophisms and to the logic of knowing, doubting, and believing.

In my contribution, I investigate the relation between the modal notion of *impossibility* and the epistemic concepts of *intelligibility* or *imaginability*. This relation, which was already advanced in several 13th-century works devoted to the analysis of impossible *positio*, survives in a revised form in some early 14th-century accounts of *positio*, such as those put forward by Walter Burley, William of Ockham, and by the two ‘Calculators’ Roger Swineshead and Thomas Bradwardine.

The second essay, by Barbara Bartocci and Stephen Read, includes a new critical edition and an English translation of an anonymous *Epitome* of John Dumbleton’s solution to *insolubilia*. This *Epitome*, preserved in a single manuscript from the Biblioteca Antoniana in Padua, includes both a survey of different types of insolubles, and a presentation of Dumbleton’s cassation-

ist solution – a solution which the anonymous author professedly adopts. However, the author of the *Epitome* does not uncritically follow Dumbleton's solution, but rather revises it quite substantially, as Bartocci and Read show.

The third essay, by Miroslav Hanke, takes its move from the approach to epistemic sophisms taken by William Heytesbury and by other Calculators in the first half of the 14th century, and then passes to examine the impact that these views had on 16th-century logical developments connected to the circle of John Mair in Paris, an impact that was probably mediated by authors like Paul of Venice and Cajetan of Thiene.

IRENE BININI

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MEDIEVAL THEORIES ON THE CONCEIVABILITY OF THE IMPOSSIBLE: A SURVEY OF IMPOSSIBLE *POSITIO* IN *ARS* *OBLIGATORIA* DURING THE 13TH-14TH CENTURIES

IRENE BININI*

Abstract: During the 13th century, several logicians in the Latin medieval tradition showed a special interest in the nature of impossibility, and in the different kinds or ‘degrees’ of impossibility that could be distinguished. This discussion resulted in an analysis of the modal concept with a fineness of grain unprecedented in earlier modal accounts. Of the several divisions of the term ‘impossible’ that were offered, one became particularly relevant in connection with the debate on *ars obligatoria* and *positio impossibilis*: the distinction between ‘intelligible’ and ‘unintelligible’ impossibilities. In this article, I consider some 13th-century tracts on obligations that provide an account of the relation between impossibility and intelligibility and discuss the inferential principles that are permissible when we reason from an impossible – but intelligible – premise. I also explore the way in which the 13th-century reflection on this topic survives, in a revised form, in some early 14th-century accounts of *positio*, namely, those of William of Ockham, Roger Swineshead and Thomas Bradwardine.

Keywords: Conceivable vs. inconceivable impossibility; *positio impossibilis*; *Tractatus Emmeranus*; William of Sherwood; Walter Burley; William of Ockham; Roger Swineshead; Thomas Bradwardine.

1. Introduction: *obligationes* and the analysis of impossibility

By the end of the 12th century several logicians in the Latin medieval tradition began to develop a special interest in logical argumentations based on impossible premises, and hence the “need for a form of consequence which

* This research has received funding from the European Union’s Horizon 2020 research and innovation programme, under the Marie Skłodowska-Curie grant agreement n° 845061.

could be used in reasoning about acknowledged impossibilities” arose.¹ This interest and need have their origin in the logical developments taking place in the first half of the century, more specifically in the ‘discovery’ of the difficulties related to the modal definition of *inferentia* and the paradoxes deriving from it, such as the principle that ‘anything follows from the impossible’. But the interest in the logical behaviour of impossible premises also stems from theological concerns, and particularly from the necessity to provide an account of doctrinal truths which are metaphysically or naturally impossible, such as the Trinity or the coexistence of human and divine nature in the same individual.

Not only were late-12th-century and 13th-century logicians interested in what validly follows from the impossible – they also paid special attention to *the nature of impossibility* itself, and to the *different kinds of impossibility* (or even, the different *degrees* of impossibility) that could be distinguished. This discussion resulted in an analysis of the modal concept with a fineness of grain unprecedented in earlier modal accounts. Several divisions of the term ‘impossible’ were proposed, such as that between *per se* and *per accidens* impossibilities; between absolute and qualified impossibilities; between the impossible in itself (*inquantum impossibile*) and the impossible derived from a union or division of terms; between syntactic and non-syntactic impossibilities; and finally – which will be the main interest of this article – between impossibilities that can be the object of understanding or belief and those that cannot, or, in other words, between what we would now call ‘conceivable’ and ‘inconceivable’ impossibilities.

The debate on the nature and taxonomy of impossibilities took place in several logical contexts in the 13th century, most prominently in the literature

1 MARTIN 2018, 354.

on *syncategoremata* and on *ars obligatoria*. To my knowledge, however, it is only in the latter that the distinction between conceivable and inconceivable impossibilities becomes particularly relevant, especially in connection with the discussion of what was called *positio impossibilis*. Impossible *positio* is a specific species of *positio* obligation in which the initial postulation, advanced by the opponent and conceded by the respondent at the start of the dispute, is an impossible statement – either a *natural* impossibility, such as ‘a man is a donkey’, or alternatively some metaphysical, doctrinal or pragmatic impossibility, like ‘an infinity exists in actuality’, ‘God is not three and one’, ‘you concede that you are dead’ or ‘Socrates ceases to know that there is nothing he ceases to know’. Not all authors agreed on the admissibility of an impossible *positum* in an obligatory dispute, but the logicians who did agree often claimed that an impossible premise could be accepted on two conditions: (i) as long as it does not entail a contradiction or anything ‘more impossible’ than the premise itself; and (ii) as long as it is an ‘intelligible impossibility’, one that is fit to be held as an opinion or entertained as a belief by a rational interlocutor. In this article, I consider some 13th-century tracts on obligations that provide an account of the relation between impossibility and intelligibility, and I examine the ideas that they offer on the nature of impossibility and the kind of inferential principles that are permissible when we reason from an impossible premise. I also explore some of the ways in which the 13th-century reflection on this topic survives, though in a revised form, in the early 14th century.

The distinction between conceivable and inconceivable impossibilities can be found in one of the earliest treatises we have available on this topic, the anonymous *Tractatus Emmeranus de impossibili positione*, which will be the subject of Section 2. The author of this tract presents some impossible state-

ments as ‘intelligible’, in the sense that we may form an understanding of one “as if it were true.”² Impossibilities like ‘God is a man’, ‘a man is a donkey’ and ‘Socrates is Brunellus’ are of this sort, and are contrasted with other kinds of impossibilities which are entirely unintelligible, such as ‘a man is not a man’ or ‘a whole has no parts’. Two later tracts on obligations – the one presumably written by Sherwood in the middle of the 13th century (analyzed in Section 3) and Burley’s composed in 1302 (Section 4) – report a similar division, namely one between *opinabiles* and *inopinabiles* impossibilities. The former are those whose falsity is not universally and immediately manifest and that may thus be entertained as the object of thought or even belief. This encompasses both physical impossibilities like ‘the Earth is greater than the Sun’, and natural or metaphysical impossibilities like, again, ‘a man is a donkey’ or ‘a man is not animal’. *Inopinabiles*, on the contrary, are impossibilities to which no rational understanding would ever assent, since their opposites’ truth is patently and *per se* known, like ‘a whole does not have parts’, ‘a good thing is a bad thing’ or ‘an animal is not an animal’.

Although the distinction between *opinabiles* and *inopinabiles* impossibilities is not to be found in later tracts, some of the ideas concerning the relation between impossibility and intelligibility developed in the 13th century do clearly underlie 14th-century theories of *positio*, such as Ockham’s theory of *ars obligatoria* included in the *Summa* (ca. 1324), Roger Swineshead’s tract on obligations (ca. 1330–5) and Thomas Bradwardine’s *De causa Dei* (1344). These 14th-century works will be considered in the final section of this article (Section 5).

2 DE RIJK 1974, 117–8.

2. *Tractatus Emmeranus de impossibili positione* and the intelligibility of some natural impossibilities

Among the oldest sources available to us that offer a discussion of impossible *positio* is the tract known as *Tractatus Emmeranus de impossibili positione* (*TEI*). This short but extremely interesting work offers both a justification of why impossible *posita* are admissible in *positio*, and an analysis of the nature of their impossibility and of the inferential principles that are applicable when reasoning from them. The tract is found in a Munich manuscript (clm 14458, f. 40^{va-vb}) that formerly belonged to the library of the monastery of Sankt Emmeran, from which its name is derived; It has been edited by De Rijk in 1974.³ It has received some attention in recent literature,⁴ where it is often discussed in parallel with the tract on *positio falsa* (*TEF*) that can be found in the same part of the manuscript and which, as De Rijk noticed, is written in the same mid-13th-century handwriting. We have no clear evidence on who the author of the two tracts was, nor when exactly they were composed. There are, however, elements indicating that the doctrines advanced in *TEI* still belong to the tradition of 12th-century Parisian logical schools: there is an explicit reference to the school of the *Parvipontani* (which the anonymous author calls *Adamiti*, as the school was sometimes called in England)⁵ and to one of the principles governing their logic for conditionals, namely that ‘anything follows from an impossible antecedent’. Moreover, the author relies on some of the key elements of Abelard’s theory of conditionals, as well as several is-

3 DE RIJK 1974, 96. Translation in YRJÖNSUURI 2001(1), 217–223. Unless otherwise specified translations of the text are taken from Yrjönsuuri.

4 Apart from DE RIJK 1974, see also MARTIN 1992; MARTIN 2001; YRJÖNSUURI 2001; MARTIN 2018.

5 See in particular DE RIJK 1974, 102. De Rijk takes this as the sign that the tract on impossible *positio*, although showing familiarity with the Parisian logical tradition, was not necessarily written in Paris, but might have been written in England, where some of the logical theories from the Continent migrated at the turn of the 13th century (*Ibid.*, 102–3).

sues and terminological features that were unique to Abelard's followers, the *Nominales*. For this reason, it has been proposed that the author of *TEI* might have belonged to the epigones of the school of the *Nominales*, and that the first years of the 13th century are the most plausible date for its composition.⁶ Even though there certainly is an Abelardian flavour to the theories advanced in this tract, it must be noted that there are also significant ways in which the author distances himself from Abelard's views, in fact reshaping some of his ideas on impossibility and entailment into a quite different paradigm, as we will see.

In the opening section of the tract, two arguments are advanced supporting the claim that (some) impossible statements can be used as *posita* in a logical disputation. The first argument establishes an analogy between the use of impossible and possible *positio*: just as we may postulate and concede a possible *positum* "in order to see what follows from it" (*ut videatur quid inde sequitur*), similarly, an impossibility may also be conceded "in order to see what would happen" (*ut videtur quid inde accidat*). This claim is based on the authority of Aristotle, who allegedly admitted that an impossibility could be used as a premise "to see what would follow."⁷ The author, however, does

6 On the connection between *Tractatus Emmeranus de positione impossibili* and Abelard's school see MARTIN 1992, 124; MARTIN 2018, 353–4.

7 DE RIJK 1974, 117–8. A similar claim may be found in another 13th-century text, the so-called *Obligationes Parisienses* (edited in DE RIJK 1975). Here, the anonymous author writes that "an impossibility must be posited in order to see what would follow from it" and attributes the idea to Aristotle (*Ibid.*, 52: "*Et hoc vult Aristoteles. Dicit enim: 'impossibile ponendum est ut videatur quid inde sequitur'*"). It is not clear to which Aristotelian passage the two authors are referring. As De Rijk writes in his edition of the *Parisienses*, the phrasing is not found in Aristotle. According to YRJÖNSUURI (2001(2), 25), this principle presents affinity with what Aristotle claims in the *Prior Analytics* on the idea of assuming a possibility "in order to see whether anything impossible follows." According to Martin, on the contrary, the textual basis for this principle is not found in Aristotle but rather in Boethius' *De Hypotheticis Syllogismis* (I, 2, 6), in which Eudemus' views are reported about those impossible hypotheses (*positiones*) that can be agreed upon "in order that reason may be pursued to its limit" – what Martin has called the "Eudemian procedure." (MARTIN 2001, 64–6)

not elaborate on this idea of ‘exploring what would happen’ once an impossibility is posited, nor does he comment further on the Aristotelian grounds supporting this idea.

The second argument advanced to justify the use of impossible *positio* is more engaging: the author claims that there are some impossibilities that we do in fact use in philosophical discourse, for instance when we say that ‘God is a man’, which is impossible by virtue of an incompatibility between the nature of *deitas* and that of *humanitas*. Not only do we use this statement as a premise, he continues, but we also *have an understanding* of what it says, meaning that we can conceive what things would be like if it were true (*nos possumus intelligere Deum esse hominem esse verum*). Analogously, we should admit that impossible statements like ‘a man is a donkey’ are similarly intelligible or conceivable. Indeed, a man being a donkey seems even more intelligible than God being a man, since humanity and deity are more ‘remote’ than *humanitas* and *asinitas* with respect to nature (*secundum naturam*).⁸

Note that the latter statement, which is often used to exemplify *positio impossibilis* in the medieval literature on the subject, is also mentioned in *TEF* as a case of *per se* impossibility, there contrasted with impossibility *per accidens*. A *per se* impossibility, in that context, is characterized in terms of an incompatibility with nature, as what can in no way be true because of a natural repugnance between the form that is predicated and the *res* which is the subject.⁹ A *per accidens* impossibility, on the contrary, is a statement that ‘be-

8 The parallel between the two cases of impossibility is not unmotivated: there is plenty of evidence of the application of impossible *positio* to doctrinal matters in theological contexts, where many natural impossibilities (the Trinity being the most patent example) are accepted as reasonable. On the use of *positio impossibilis* in theological contexts see MARTIN 2001; YRJÖNSUURI 2000; KNUUTTILA 1997.

9 DE RIJK 1974, 113, 12-5: “Impossibile per se est illud quod nullo modo potest esse verum, quando scilicet forma predicati naturaliter repugnat rei subiecti, sicut hoc ‘homo est asinus’.”

comes' impossible only by virtue of a certain determination added to it (*respectu alicuius determinationis*), like the temporal qualification 'in this instant' in 'It is impossible for Socrates to be white in this instance'.¹⁰ Assuming that the two tracts were written by the same author, we might infer that he maintains there are *per se* or 'absolute' (namely, non-qualified) impossibilities that are nevertheless intelligible.

Another intelligible impossibility that the author considers in *TEI* – which again he claims to be derived from Aristotle, although from which passage is not clear – is the scenario in which a fish is removed from the water in such a way that nothing else assumes its place. This is a different form of impossibility than the ones mentioned above, not consisting in the predication of naturally incompatible terms but rather in the violation of a law of physics, the impossibility of a void.¹¹ Despite it being physically impossible, the author again claims that we can have an understanding of this scenario, just as we can conceive of a man being a donkey or God being a man. It is indeed the intelligibility of such situations that ensures the admissibility of the corresponding statements as *posita*: the author repeatedly observes that any statement, possible or impossible, can be used in *positio* as long as it is conceivable. This is because “when we can understand, we can posit, and thus concede” (*Et ita cum possumus intelligere, possumus ponere, et ita concedere*).¹²

From the fact that some impossible statements are intelligible and thus

10 Although we rarely encounter this distinction between modalities *per se* and *per accidens* in the 12th century, the distinction proposed here strongly resembles the distinction between 'absolute' and 'qualified' or 'determinate' modalities that was advanced by Abelard and other 12th-century logicians. See on this BININI 2021, 33–44; 177–182.

11 For this interpretation of the passage see YRJÖNSUURI 2000, 59.

12 DE RIJK 1974, 118, 2. See also *Ibid.*, 118, 7–8: “Since we can posit that which we can understand, it is clear that an impossible *positio* must be admitted and an impossible conceded” (*cum possumus ponere illud quod possumus intelligere, patet quod impossibile positio est recipienda et impossibile est concedendum*). Both translations are from YRJÖNSUURI 2001(1), 217.

positable, however, it does not follow that any impossibility whatsoever should be admitted. The author stresses that not all statements including “two terms that are opposite in a contradictory way”¹³ (*duo contradictorie opposita*) should be used in *positio impossibilis*. The kind of impossibility that is brought about by the predication of contradictory opposites, as in ‘a man is not a man’, is thus seen as a different and more problematic impossibility than the one derived from the predication of physical or metaphysical incompatibles, like ‘man’ and ‘donkey’ or even ‘man’ and ‘God’. Statements predicating such contradictory impossibilities seem to be entirely *unintelligible*, perhaps because we could not form an understanding of what things would be like if they were true.

Other given examples of unintelligible impossibilities are statements that do not contain but nevertheless entail a predication of contradictory opposites. The *positum* ‘a man exists necessarily’, which is not unintelligible *per se*, becomes so when we specify that ‘being mortal’ is part of the definition of man. Adding this to the *positum* would lead to a predication of contradictory opposites (‘a man can and cannot die’), so that “this kind of impossible statement cannot be posited in any way” (*tale impossibile nullo modo potest poni*).¹⁴ This shows us that the intelligibility and admissibility of a given statement may be context-sensitive, depending on which other claims are used as assumptions. Another impossible statement which is ruled out as entirely inadmissible and unintelligible is the paradoxical claim ‘Socrates ceases to know that there is nothing he ceases to know’ which, as the author says, “*nullo modo potest poni*,” again for the reason that a predication of two contradictory opposites could be derived from it.

An objection is raised at this point: shouldn’t the statement ‘a man is a

13 DE RIJK 1974, 118, 14–5.

14 *Ibid.*, 119, 29–30.

donkey' be ruled out as inadmissible on the same grounds, because of the risk of entailing a contradiction? Indeed, if we take the *inferentia* (*) 'if something is a man, it is not a donkey' as valid, one could use it to derive that 'something which is a donkey is not a donkey', which is an unintelligible impossibility. And such an inference does seem valid because, as the author states, the consequent follows from the antecedent by virtue of nature (*naturaliter sequitur*). In answer to this objection, the author of *TEI* offers his views on the specific kind of *consequentia* that is permitted in the context of *positio impossibilis*. He says that inferences like (*) are not acceptable in this context because, just as we posit and concede *posita* not with respect to *what is possible in nature* but with respect to *what is intelligible or conceivable*, similarly we must evaluate consequences – and the inseparability relationship between things that a good *consequentia* is supposed to represent – not *secundum naturam*, but rather *quantum ad intellectum*, with respect to what can be conceived as united or separated. Because being a man and being a donkey, though naturally incompatible, can be understood as united in the same subject, it does not follow *quantum ad intellectum* that if something is a man, it is not a donkey. More generally, the author concludes, any consequence in which a negation follows from an affirmation is not permitted, because it would not follow with respect to the understanding.¹⁵

As Martin has pointed out, the latter principle to which the author appeals (that a negation cannot validly follow from an affirmation) was one of the distinguishing principles of Abelard's theory of conditionals (and, more

¹⁵ *Ibid.*, 119, 8–15: "Solutio. Cum impossibilis positio non habeat fieri respectu nature sed quantum ad intellectum, cum ille due forme non possint esse in eodem subiecto naturaliter, bene sequitur quantum ad naturam: 'si est homo, non est asinus'. Sed quia potest intelligi quod ille due forme sint in subiecto, quantum ad intellectum non sequitur. Unde cum impossibilis positio habeat quantum ad intellectum, patet quod in impossibili positione non debet concedi consequentia in qua negativa sequitur ex affirmatione."

specifically, that the *locus* from opposites is not an acceptable ground for a good *inferentia*). Also of Abelardian origin is another criterion that the author mentions, namely, that only consequences in which “the understanding of the consequent is contained in the understanding of the antecedent” (*intellectus consequentis clauditur in intellectu antecedentis*) are admissible in *positio impossibilis*.¹⁶ This clearly echoes what has been called Abelard’s ‘containment criterion’ for the truth of conditionals, expressed in the *Dialectica* as requiring, in order for an *inferentia* to be valid, that the *sensus* of the consequent be already contained in the *sensus* of the antecedent, so that the antecedent “by itself requires the consequent.”¹⁷ The author of *TEI* refers to these as *consequentiae rectae*.

Nevertheless, there are also some aspects of the view presented in *TEI* that are not found in Abelard, and indeed seem to go partly against his theory of conditionals. For one thing, the author stresses that the ‘containment’ which is supposed to be present between consequent and antecedent is purely *epistemic*, as the containment of an understanding in another understanding (*intellectus in intellectu*). Abelard, on the other hand, thought of the

16 *Ibid.*, 118, 17–22: “Preterea. Notandum quod eadem est ars falsi positionis et impossibilis positionis. Unde notandum quod sicuti in falsi positione omne hoc quod sequitur ex positio, est concedendum, sic in impossibili positione omne sequens ex positio est concedendum; ‘sequens’ dicitur secundum rectam consequentiam. Et est recta consequentia quando scilicet intellectus consequentis clauditur in intellectu antecedentis.” The author returns to the same point a few lines later, writing that: “Sed tantummodo illa consequentia est concedenda in hac questione in qua intellectus consequentis clauditur in intellectu antecedentis.” As has been noted, this criterion is the reason why the author excludes the principle *ex impossibili quodlibet sequitur* from impossible *positio*.

17 On Abelard’s criterion see in particular MARTIN 2004; MARTIN 2018. Note, however, that the terminology used by the author of *Tractatus Emmeranus* for his criterion – of an *intellectus* being included (*clauditur*) in another *intellectus* – is not exactly the same as the one that Abelard used in his *Dialectica*, where he rather speaks of the sense (*sensus*) of the antecedent requiring (*exigere*) that of the consequent, or alternatively of the sense of the consequent being contained (*continere*) in that of the antecedent. We find terminology similar to that of *Tractatus Emmeranus* in another 13th-century tract on obligations, which is attributed to Nicholas of Paris. Here, a good natural consequence is said to be “*cum consequens in antecedenti clauditur*,” see BRAAKHUIS 1998, 69, 27–8.

containment between antecedent and consequent as a metaphysical, semantic and epistemic relation: for him, an *inferentia* like ‘if something is a man, it is an animal’ is good and necessary because (i) being an animal is a property that is part of the nature of man; (ii) ‘animal’ is included in the meaning of ‘man’; and finally, (iii) the understanding (*intellectus*) of ‘animal’ is included in that of ‘man’.¹⁸ Even though the metaphysical relation expressed in (i) is the ultimate source for the validity of the inference – for Abelard takes the nature of things as the *vis inferentiae* of all good (non-formal) conditionals – he stills seems to use (i)–(iii) as if they were interchangeable. The reason for this might be that, according to Abelard, the domain of *intelligibility* is entirely coincident with the domain of *metaphysical possibility* – in the sense that what can be conceived coincides with what is compatible with the nature of things.¹⁹ The congruity between the domain of nature and the domain of *intellectus* allows him to pass from talking about metaphysical to talking about epistemic relations in a way that the author of *Tractatus Emmeranus* seems to find unacceptable. For him two things or two forms may be inseparable with respect to nature but perfectly separable in the intellect and, conversely, the relation of metaphysical containment between one substance and its essential forms may have no correspondent on the level of the understanding – one may be able to intellectually separate a human from their rationality or mortality, or to unite being man and being donkey in the same understanding. It does not seem to me that our author has misunderstood Abelard on this point, as Martin has suggested,²⁰ but rather that he has lost confidence in something that Abelard took for granted: that our ways of understanding and

18 There is some evidence, though, that when Abelard speaks of the containment of a *sensus* in a *sensus*, what he means is in fact the containment of an *intellectus* in another *intellectus*. Although he is not very explicit on this in the *Dialectica*, this is remarked at least once in his *De Intellectibus*: cf. MARTIN 2004, 183 on this point.

19 On this see CAMERON 2020, *passim*, and BININI 2021, p. 211–2.

20 MARTIN 2001, 67.

conceiving things necessarily mirror and replicate the way in which things naturally are. In his view, there is a mismatch between what is naturally (in)compatible and what is intellectually (im)possible, and this discrepancy is reflected in the two different ways of defining (im)possibility: according to nature and according to the intellect.

Just as the inseparability between things can be considered with respect to nature or with respect to the intellect, so the relation of following – which represents such inseparability – can be distinguished as being valid *respectu naturae* or *quantum ad intellectum*. *Tractatus Emmeranus*, then, showcases what we may define as an ‘epistemic turn’ in the definition of *consequentia* and in the modal ideas of (in)compatibility or inseparability between things to which the notion of following was traditionally associated. This epistemic definition does not replace the nature-based account that was given by Abelard and others in the 12th century, but is proposed by the author as an alternative to it, and the two criteria for modality and consequence are said to be applicable in different contexts: the naturalistic one when we deal with possible postulations, the epistemic one when we reason starting from an impossible *positum*.

As we will see, the author of *TEI* is not the only author in the 13th century to relate modalities and *consequentiae* to psychological or epistemic notions. Indeed, the association between the notion of impossibility and the domain of intelligibility or conceivability was also acknowledged and suggested by others, especially in connection with impossible *positio* and, more generally, with the idea of reasoning from impossible premises. Some of these views will be the object of the next section.

3. William of Sherwood(?) on 'credible' and 'incredible' impossibilities

Another 13th-century tract containing an analysis of impossible *positio* is the *De Obligationibus* attributed to William of Sherwood, which Romuald Green provisionally edited in 1963.²¹ Four manuscripts preserve this work: (i) Paris, B.N. Ms. lat. 16617; (ii) Venice, San Marco, Ms. X 204 (Z.L. 302); (iii) Erfurt, Amplon, Ms. 4" 259; and (iv) Paris, B.N. Ms. lat. 16130.²² The tract is particularly interesting because it is the only one, among the tracts on *ars obligatoria* written in the mid 13th century, to include a separate discussion of impossible *positio* and of the inferential rules that are admissible in such disputational contexts. Other works on obligations dated to the same period – like the *Obligationes Parisienses*,²³ the *De Petitionibus Contrariorum*²⁴ and Nicholas of Paris's *Obligationes*²⁵ – only seem to take false but possible statements as *posita*, and do not discuss *positio impossibilis* as such, even though they do offer views on the nature of impossible propositions and their logical behaviour, as we will see. It is significant, then, that Sherwood claims that at least some impossibilities are admissible in a *positio* dispute. Also significant is the reason offered to justify their admissibility, which echoes the one advanced in the earlier *Tractatus Emmeranus*: since some impossibilities are intelligible and may even constitute the object of an interlocutor's belief, Sherwood maintains, they should be conceded as premises in a disputation.

Before proceeding with the analysis of this text, something should be said on its paternity, which has been the object of some controversy. Green

21 GREEN 1963.

22 For a description of these manuscripts and their interrelation see SPADE, STUMP 1983, 11 ff.

23 DE RIJK 1975.

24 DE RIJK 1976. The text is preserved in the same manuscript that also contains Sherwood's (?) *De Obligationibus*: Ms. Paris, B. N. lat. 16. 617, f. 64v.

25 Preserved in Ms Paris, B. N. lat., 11.412 and edited in BRAAKHUIS 1998.

was the first to present the attribution to William of Sherwood as doubtful,²⁶ and Spade and Stump then added further reasons to object to it,²⁷ proposing that the treatise should rather be dated to a much later period, being written perhaps by Burley or by one of his contemporaries at the turn of the 14th century. This they argued by highlighting the level of sophistication of the theory advanced in the text (particularly, its appeal to a rather complex taxonomy of *consequentiae*), in addition to further reasons. More recently, however, other researchers put the question of paternity back on the table, presenting convincing arguments in favour of the attribution to Sherwood and thus dating the text to 1240–1260. They objected to the attribution to Burley by pointing out that the level of doctrinal sophistication that is found in these *obligationes* is in fact entirely compatible with the logical developments in the middle of the 13th century.²⁸ In addition, Braakhuis identified another tract on *ars obligatoria*, which he attributed to Nicholas of Paris and dated to 1230–1250, and which presents many doctrinal and terminological similarities to the tract attributed to Sherwood, suggesting that the latter was likely written around the same time.²⁹ Although none of these studies settle the question definitively, I do find the evidence provided by Martin, Braakhuis and others very compelling. If we also reconsider this question in light of the recent studies which have been done on mid-13th-century theories of *consequentiae*,³⁰ we get the clear impression that the views advanced in our tract are perfectly in tune with the logical climate in which Sherwood wrote and taught. Therefore, I will here refer to this tract as the product of William of Sherwood, and shall compare it with his *Syncategoremata* and his *Introductiones in Logicam*.

26 GREEN 1963; but see DE RIJK 1976, 28, note 11 questioning Green's reasons for doubting the attribution to Sherwood.

27 SPADE, STUMP 1983, *passim*.

28 D'ORS 1990; MARTIN 2001; BRAAKHUIS 1998; and VOS 2008.

29 BRAAKHUIS 1998, 152–3.

30 See in particular SPRUYT 2018.

According to Sherwood, all *positio* disputes start from a false *positum*. False *posita* are divided into two categories: possible and impossible ones, both of which are acceptable in *positio*. For impossible *posita*, however, the author puts forward two questions: (i) whether any impossibility whatsoever may be posited; and (ii) according to which inferential principles should the reasoning proceed once an impossibility is conceded? In answer to the first question, Sherwood observes that a *positum* always stands for an opinion (*opinio*), namely, for the object of thought or belief that a philosophical interlocutor may entertain in her mind, and based on which a dialectical dispute could be launched. If all *posita* are meant to represent an *opinio*, the author continues, it follows that only those impossibilities are admissible that can be entertained as the object of thought or belief (*quae possunt opinari*) – but not all impossibilities are of this sort. Impossibilities may indeed be divided into ‘credible’ and ‘incredible’ ones (*opinabiles vs inopinabiles*). Although Sherwood does not define the term (*in*)*opinabile*, in medieval logic and rhetoric this word was often used – along with others such as *probabilis*, *credibilis*, and *verisimilis* – to indicate an “eligible opinion,” namely “an opinion (or proposition) [that] was not only *prima facie* adoptable, but also fit to be held as true.”³¹

The distinction between *opinabiles* and *inopinabiles* impossibilities that Sherwood is presenting here – which is also found in Burley’s tract on obligations, as we will see in Section 4 – resembles the one between ‘intelligible’ and ‘unintelligible’ impossibilities that was evoked in *Tractatus Emmeranus*, where the anonymous author also presented the domain of ‘positability’ as coincid-

31 SCHUESSLER 2019, 37. In chapter 1 of this book, Schuessler argues that the terms ‘*opinabilis*’, ‘*credibilis*’ and ‘*probabilis*’ were used as synonymous either in the sense of ‘reputable’, ‘probable’ or ‘approvable’ opinion, or also in the sense that a certain proposition was “fit for adoption and sufficiently, although not optimally, backed by reasons for truth” (*Ibid.*, 37–8). Being fit for adoption in a dispute, i.e., being such that it could be held as true, seems to be exactly the meaning that Sherwood has in mind here.

ent with that of ‘intelligibility’. However, whereas in *Tractatus Emmeranus* the impossibilities categorized as ‘unintelligible’ were associated with the notion of contradiction and the predication of contradictory opposites (the author claimed that unintelligible statements are those that include or entail *duo contradictorie opposita*), Sherwood outlines the distinction between *opinabiles* and *inopinabiles* impossibilities not in strictly logical terms, but rather in epistemic ones. An ‘incredible’ impossibility is defined as the opposite of a statement the truth of which is certain and immediately manifest to everyone. Statements like ‘every whole is greater than its parts’, for instance, are evident in such a way that anyone, upon hearing them, believes that they are true. The opposites of these claims, thus, cannot be entertained as beliefs by any rational agent. On the other hand there are statements whose truth, despite being necessary, is hidden and not universally accessible, such as ‘the Sun is greater than the Earth’. The opposites of these claims are impossibilities that *can* be understood or believed, and as such can also be used in *positio*.³²

In his tract, Sherwood gives us a few examples of incredible impossibilities: apart from the aforementioned (i) ‘every whole is not greater than its parts’, (ii) ‘a good thing is a bad thing’ and (iii) ‘a man is not a man’ are also presented as entirely *inopinabiles*, and thus unusable as *posita*. These are contrasted with other predications of natural opposites, such as (iv) ‘a man is a donkey’ and (v) ‘a man is not an animal’, which the author rather takes as intelligible and credible, and therefore admissible in *positio*. It is important to

32 See GREEN 1963, 24: “Habito de positione possibili, sequitur de impossibili, et primo videndum est an quodlibet impossibile possit poni. Secundo, qualiter procedendum est in tali positione. Cum igitur positio est opinio, impossibile quod non potest opinari, non potest poni ut patet, et huius sunt aliqua. Sunt enim quaedam quae ita sunt vera et manifesta, quae, mox audita, sunt manifesta, quale est hoc quod opinatur: ‘omne totum est maius sua parte’. Et horum opposita nullo modo possunt opinari nec poni. Iterum, sunt alia latentia, quorum opposita possunt opinari, et huius possunt poni.” [I have slightly changed Green’s punctuation.]

notice that contradictory statements like (iii), which we would now categorize as *syntactic* or *formal impossibilities*, are included in the domain of unbelievability but do not exhaust it. The reason for their incredibility seems unrelated to their structural or syntactic features. Nor do incredible impossibilities appear to coincide with *natural* or *metaphysical* impossibilities, otherwise statements like (iv) and (v) would be the most plausible candidates. On the contrary, Sherwood suggests that (i)-(iii)'s being *inopinabiles* depends on something to do with our epistemic faculties: these propositions' falsity, he claims, is so beyond doubt that no rational interlocutor may ever entertain them in her mind and believe that they are true, since the truth of their opposites is immediately obvious, as was said. Sherwood's claim that the truth of statements like 'a whole is greater than its parts' is such that, "as soon as we hear them" (*mox audita*), we believe that it is true seems to mean that our knowledge of these claims is not the product of a process of reasoning, and is thus not mediated by our knowledge of other claims.

We may think that statements like (iv) 'a man is a donkey' or (v) 'a man is not an animal' are also immediately and manifestly false, and thus unintelligible. One reason in favour of this view is their similarity to statements like (ii) 'a good thing is a bad thing', which Sherwood holds as 'incredible' on the basis of the authority of Aristotle. Another reason is that believing that 'a man is a donkey' or 'a man is not an animal' seems to be, at least at first sight, the same as believing that (iii) 'a man is not a man' or (vi) 'an animal not an animal', which are "even more incredible" (*multo fortius non potest opinari*) than (ii). Moreover, just as the author of *Tractatus Emmeranus*, Sherwood also thinks that predications like 'a man is a donkey' are *per se* impossibilities, that is to say, they are impossible in every moment of time and in an absolute

sense.³³ Notwithstanding these reasons, Sherwood argues that metaphysical impossibilities like (iv) and (v) are in fact believable, unlike (ii) or (iii). This has to do, once again, with the way in which we know things, rather than with these things' natures or with the formal structure of a statement: even though we could never intellectually separate *being a man* from *not being a man*, nor reasonably conceive *being good* and *being bad* as united in the same thing (as the knowledge of one is intrinsically connected to the knowledge of the other), it is nevertheless possible to conceive *being a man* and *being a don-*

33 Although Sherwood does not elaborate on the distinction between *per se* and *per accidens* modalities in this tract (he uses this terminology in his discussion of *positio*, but never defines it), he does provide a characterization of these concepts elsewhere, in both his *Introductiones in Logicam* and his *Syncategoremata*. Not dissimilarly from his contemporaries, Sherwood thinks that a *per se* impossibility, which he also labels 'absolute', is an impossibility with respect to any moment of time, either past, present, or future; a *per accidens* impossibility is one that cannot be true with respect to the present or the future, but could have been true in the past. Examples offered for the latter kind of impossibility are past statements like 'I did not walk', whereas 'a man is a donkey' is the standard example for *per se* or absolute impossibility. See e.g. SHERWOOD 1983, 232, 37–41: "Et sciendum, quod impossibile dicitur duobus modis: (1) uno modo, quod non potest nec poterit nec potuit esse verum, et est impossibile per se, ut: 'Homo est asinus'. (2) Alio modo, quod non potest nec poterit esse verum, potuit tamen, ut cum dicam: 'Ego non ambulavi'. Et est impossibile per accidens." See also Sherwood's *Syncategoremata* (Sherwood 2012, 152) where the distinction between absolute and *per accidens* impossibility is discussed in the section devoted to the syncategorema *si* and in connection with the distinction between natural and nonnatural consequences. It is worth noting here that the way in which Sherwood presents the distinction between two kinds of impossibility, in purely temporal terms, slightly differs from the characterization of the same distinction given in *TEF*, where the author presents *per accidens* impossibility as a generally 'qualified' form of impossibility, as impossibility "respectu alicuius determinationis," in contrast to an absolute or unqualified one. For *Tractatus Emmeranus*, temporal determinations are just one of the several determinations that may qualify impossibility, thus rendering it *per accidens*. Moreover, *Tractatus Emmeranus* presented absolute impossibility not simply as the sempiternal impossibility of being true but rather as general "incompatibility with the nature of things." These features indicate that *Tractatus Emmeranus* more closely resembles 12th-century Parisian logic, whereas the characterization of *per se/per accidens* modalities that is offered by Sherwood is in line with the later, mid-13th-century way of categorizing modalities. We find a distinction between *per se* (or *simpliciter*) and *per accidens* impossibilities similar to that of Sherwood in both Nicholas of Paris' *Obligationes* (BRAAKHUIS 1998, 165 and 189, 31–7) and in *Obligationes Parisienses* (DE RIJK 1975, 32, 24–28). In both 'a man is a donkey' exemplifies *per se* impossibility.

key as united in one subject, or *being man* and *being animal* as separated. This may happen if the knowledge we have of humans is imperfect and not *per se*.³⁴ Again, what Sherwood says on this point echoes an idea that was already present in *Tractatus Emmeranus*, even though less emphasis was placed on the notion of knowledge in the earlier tract: the two texts are similar in that they both distinguish (in)separability *at the level of nature* from (in)separability *at the level of the intellect*. The distinction that both treatises advance between intelligible and unintelligible (or credible and incredible) impossibilities is aimed at capturing this mismatch between nature and our understanding of things.

Just as in *Tractatus Emmeranus*, the admissibility of statements like ‘a man is a donkey’ or ‘a man is not an animal’ as *posita* has repercussions for the kind of inferential principles that are accepted in *positio impossibilis*. As one might know, Sherwood denies that *consequentiae infinitae* can be used in this disputational context, meaning by this the two principles according to which ‘anything follows from the impossible’ and ‘the necessary follows from anything’.³⁵ In doing so he is once again on the same page as the author of *Tractatus Emmeranus*, who also denied the validity of the *ex impossibili* principle in *positio impossibilis*.³⁶ Sherwood thinks that in a *positio* of any sort, both

34 GREEN 1963, 25, 11–22: “Contingit enim cognoscere hominem imperfecte cognoscendo ex quibus est secundum naturam, ut materiam et formam suam, vel ex quibus secundum rationem, ut genus et differentiam. Cognoscens ergo hominem primo modo solum potest opinari hominem non esse animal, nec est idem quod opinari animal non esse animal. Contingit etiam opinari hominem esse asinum, vel primo modo vel secundo modo cognoscendo hominem; asinum, tamen, cognoscendo opposito modo. Et sic ista possunt poni. Bonum, tamen, esse malum non potest opinari, quia quantum habet aliquis de cognitione boni, tantum habet de cognitione mali, cum malum sit privatio et recessus a bono.”

35 In the *Synkategoremata*, consequences of this sort were labelled as *innaturales*, as opposed to natural consequences. The same distinction may be found in Nicholas of Paris, and other 13th-century logicians advance similar divisions. Cf. SPRUYT 2018, 337.

36 The *Tractatus Emmeranus*, however, only spoke of the principle concerning impossible antecedents, without taking into consideration the case of necessary consequents. Nich-

possible and impossible, only *consequentiae finitae* are permitted, and these include two kinds of inference: those in which the antecedent *cannot be true without* the consequent, like: ‘if something is a man, it is able to laugh’ or ‘if something is a man, it is not a donkey’; and those in which the consequent is ‘understood’ (*intelligitur*) in the antecedent, as e.g. ‘if something is a man, it is an animal’.³⁷

These examples may sound puzzling, given what Sherwood had said so far on the admissibility of statements like ‘a man is a donkey’ or ‘a man is not an animal’ as the starting point of a *positio* argument. For these premises, in addition to the inferential principles just mentioned, would result in a very short and uninteresting dispute, since the conjunction of the *positum* ‘a man is a donkey’ with the inference ‘if something is a man, it is not a donkey’ would immediately lead to a contradiction and, consequently, to a termination of the dispute. What Sherwood adds, however, is that the validity of the aforemen-

olas of Paris, who wrote a tract on obligations presumably in the same time period as Sherwood did, also denied the validity of the two principles ‘anything follows from the impossible’ and ‘the necessary follows from anything’ in *positio* disputations in which impossibilities or necessities *per accidens* are conceded. See BRAAKHUIS 1998, 220, 25–31: “Quod concedimus de necessario per accidens, sicut impossibile per accidens potest poni, dicentes quod in his obligationibus attendende sunt tantummodo antecedentia et consequentia naturalis, scilicet cum consequens in antecedenti clauditur, et cum antecedens sine consequenti esse non potest, et cum consequens de necessitate est, si antecedens est; et ita non habebit(?) hic locum illa regula: ‘ex impossibili quiclibet’, ‘necessarium <ad> quiclibet sequitur’.”

³⁷ GREEN 1963, 26, 11–24: “Quaeritur quae consequentia attendenda est in hac positione. Est enim consequentia duplex: aut finita, aut infinita. Infinita dupliciter: aut ex parte ante, qua dicitur quod necessarium sequitur ad quodlibet, aut ex parte post, qua dicitur quod ex impossibili sequitur quodlibet. Neutra istarum est hic attendenda, tum, quia infinita, et ob hoc, extra artem, cum, quia sic omnia essent sequentia, et sic non esset hic meta. Finita autem dupliciter est: quando consequens intelligitur in antecedente, et quando consequens non potest esse verum sine antecedente, cum non intelligitur in ipso. Exemplum primae: ‘si homo est, animal est’. Exemplum secundae est: ‘si homo est, risibile est’, vel: ‘si Socrates est homo, non est asinus’.” In this respect, the theory advanced by Sherwood differs from that put forward in *Tractatus Emmeranus*, which only admitted the latter *consequentiae*, those based on containment, as valid in impossible *positio*.

tioned inferential principles is somehow context-sensitive, in the sense that it depends on the premises of the disputation itself. If one posits, at the beginning of the dispute, that a man is not an animal, and this is conceded by the opponent as positable (and thus, as credible), then the *ratio* by virtue of which the inference ‘if something is a man, it is an animal’ was supposed to be valid – namely, the fact that being animal *is contained* in being man – is “destroyed” by the *positio*, because by positing such a statement we are agreeing upon the fact that being an animal is in fact not contained in the understanding of being a man (“*ponimus quod ‘animal’ non sit in intellectu ‘hominis’*”).³⁸

The same is the case for other impossible *posita*, like ‘a man is a donkey’, whose use in *positio* invalidates some inferential principles – for instance the consequence ‘if something is man, it is not donkey’ – by “destroying the *ratio*” on which those inferences were based. What Sherwood says on this point clarifies that like the author of *Tractatus Emmeranus* he, too, intends the containment at the basis of the validity of consequences as an *epistemic containment*, of one *intellectus* in another.³⁹ Unlike metaphysical containment, which should be invariable and independent of our ways of understanding and speaking of things, the epistemic containment that Sherwood has in mind here is context-dependent in various ways: it may depend on the agents participating in the disputation, on the knowledge that they have of the things under discussion (and thus on what they count as believable), or on which

³⁸ *Ibid.*, 27, 4–14: “Ad alia dicendum quod reliquae duae consequentiae [namely, ‘*si est homo, est animal*’ and ‘*si est homo, non est asinus*’] sunt sustinendae in omnibus positionibus, in quibus causa et ratio istarum consequentiarum non destruuntur ex positione. Et propterea, si ponamus hominem non esse animal, ponimus quod ‘animal’ non sit in intellectu ‘hominis’, et sic destruimus causam praedictae consequentiae. Item, si ponamus hominem esse asinum, ponimus per consequens hominem et asinum non esse opposite, et sic destruimus causam praedicate consequentiae. In quibuscumque, ergo, istarum consequentiarum causae non destruuntur ex positione, in his sunt sustinendae haec consequentiae, in aliis autem non.”

³⁹ For this interpretation see also YRJÖNSUURI 1990, who speaks of ‘conceptual containment’ in relation to Sherwood’s theory of obligations.

other postulations are made at the beginning of the dispute. Consequently, the inferential principles that are admitted in this sort of dispute are similarly context- (and maybe even agent-)dependent.

4. Impossibility and intelligibility in the early 14th century: the *De Obligationibus* of Walter Burley

The distinction between *opinabiles* and *inopinabiles* impossibilities can also be found in the treatise on obligations composed by Walter Burley in 1302.⁴⁰ As Sherwood before him, Burley points out at the beginning of his discussion of *positio impossibilis* that one cannot posit any impossibility whatsoever in disputes of this sort, but only those impossibilities that some have called ‘credible’ (*opinabiles*). Statements like ‘a man is not an animal’ are of this sort, as opposed to others like ‘an animal is not an animal’. Just as Sherwood and the author of *Tractatus Emmeranus*, Burley points out that when a credible impossibility is used in *positio*, the inferential rules that we admit must be restrained: he discards the two principles *ex impossibili sequitur quodlibet* and *necessarium sequitur ad quodlibet* as invalid, for otherwise any statement would both follow and contradict the same *positum*, and thus there would be nothing irrelevant to it.⁴¹ Burley then claims that the consequences which are

40 For a provisional edition and a description of this text, see GREEN 1963. The discussion of *positio impossibilis* is found on pp. 83–4.

41 See *Ibid.*, 83, 10–26: “Sequitur de positione impossibili, et est positio impossibilis quando propositio impossibilis ponitur. Et ideo, in hac positione nihil debet poni nisi impossibile. Non tamen est quodlibet impossibile ponendum, quia impossibile formaliter includens opposita non debet hic poni. Quia, si poneretur, repugnans posito esset concedendum, quia repugnans posito esset consequens ad positum. Et ideo, solum impossibile non includens opposita formaliter debet hic poni. Et quidam dicunt quod solum impossibile opinabile debet hic poni. Et sciendum quod in hac positione non sunt istae regulae sustinendae: ex impossibili sequitur quodlibet; necessarium sequitur ad quodlibet. Nec debet in hac positione sustineri aliqua consequentia infinita, quia si consequentia infinita esset hic sustinenda, posito impossibili quodlibet esset concedendum quia sequens, et quodlibet esset negandum quia repugnans; nam, si quodlibet sequatur, quod-

properly applicable when reasoning from an impossibility are the ‘natural’ ones, which he opposes to *consequentiae infinitae* (or, elsewhere, to ‘accidental’ consequences). Although he does not elaborate on the idea of *consequentia naturalis* in this context, in other writings of his natural consequences are defined in terms of the epistemic or conceptual containment that we have come across in earlier 13th-century texts on impossible *positio*. In his *De Consequentibus*, for instance, Burley says that “*consequentia naturalis est quando consequens est de intellectu antecedentis*,”⁴² using an expression that, as Read has noticed,

pervade[s] English treatments of consequences in the fourteenth century, in particular in the works of Richard Billingham, Robert Eland, Ralph Strode, Richard Lavenham and others.⁴³

Even the validity of natural consequences, though, should be further limited in *positio impossibilis* according to Burley. Only those *consequentiae naturales* are permitted whose truth is manifest and indubitable to everyone, which is the case when the opposites of these consequences cannot be conceived or believed. The natural consequence ‘if something is a man, it is an animal’ is not of this sort, since the opposite of what it says (a man not being an animal) is taken as conceivable or believable. On the contrary, consequences like ‘if this

libet repugnat, et ita esset concedendum et negandum. Et praeter hic, nihil esset impertinens posito.” [I have slightly changed Green’s punctuation.]

42 See GREEN-PEDERSEN 1981, 128, par. 70. For Burley’s theory of consequences see D’ORS 1990 and ARCHAMBAULT 2018. See also BOSMAN 2018, 225–7 for a brief introduction to the interpretation of the ‘containment criterion’ in this tradition, and the literature quoted there (BOH 2000, NORMORE 1993, DUTILH NOVAES 2007) for the epistemic reading of this criterion.

43 READ 2020, 283–4. Read provides a survey of the criteria for the validity of consequences in the 14th-century Oxonian tradition, and a comparison with the Parisian tradition on the same issue. For this analysis of treatments of consequences in late medieval England, Read refers to WEBER 2003.

is a whole, it has parts', or 'if this is a whole, it is bigger than each of its parts' are such that they cannot be put into doubt, and so preserve their validity even when we reason from impossible premises. Thus Burley seems to be following a quite established tradition in characterizing both the notion of impossibility and that of reasoning from an impossibility by appealing to the epistemic or doxastic notions of conceivability, believability, and the distinction of evident vs dubitable truth.

Yet two ideas that emerge during his discussion of impossible *positio* seem to be rather innovative in comparison to the tracts discussed in Sections 2 and 3 above. One is Burley's identification of credible and incredible impossibilities as being different 'degrees' of impossibility: statements like 'an animal is not an animal' are said to be "more impossible" (*magis impossibile*) than, e.g., 'a man is not an animal', even though both are equally impossible with respect to the nature of things. This idea of a 'gradability' of modalities, and of impossibility in particular, is not extraneous to Burley's modal thought. In his *De Puritate Artis Logicae* (the shorter version, written in the 1320s), when discussing the validity of *ex impossibili sequitur quodlibet*, Burley says that such a principle, so formulated, is too coarse-grained: there are in fact different sorts of impossibility, and even though he admits that both the contingent and the necessary follow from an impossible antecedent, it would still be improper to say that the more impossible validly follows from the less impossible. These degrees of impossibility are connected, in that context, to degrees of *apparency* or *evidentness*, whereas in the longer version of *De Puritate* Burley speaks of degrees of truth. Burley makes a similar point in his *quaestio* 'Utrum contradictio sit maxima oppositio', where he again denies that a *minus impossibilis* may follow from something more impossible, and adds that the relation between the less and the more impossible is analogous to that

between the possible and the impossible, that is to say, with respect to the more impossible the less impossible "*habet rationem possibilis*."⁴⁴

Another important innovation of Burley's is his characterization of the distinction between credible and incredible impossibilities in purely syntactic terms, as a distinction between statements that are 'formally impossible', in that they include a predication of terms that are formally opposites (e.g., 'animal' and 'non-animal') and those that include opposites which are incompatible but not formally so (e.g., 'man' and 'non-animal'). Burley equates the idea of formal impossibility to that of repugnancy. Sherwood, as we saw, seemed to have a wider notion of *inopinabilitas*, which included formally contradictory statements but also impossibilities of other sorts: 'a whole is not greater than its parts' and 'a good thing is a bad thing', for instance, counted as entirely *inopinabile* for him. Like Burley, the author of *Tractatus Emmeranus* also characterized the unintelligibility of statements in terms of a predication of contradictory opposites, but did not make clear whether this opposition was conceived in purely formal or syntactic terms. For Burley, on the contrary, the domain of credibility or conceivability seems to coincide with that of syntactic possibility.

Interestingly, this tract on obligations is not the only text in which Burley connects the notion of *propositio opinabilis* with the idea of syntactic or formal possibility. In one of his treatises on Aristotle's *Physics*,⁴⁵ Burley defines as *opinabilis* any proposition which "does not include a formal contra-

44 DE RIJK 1996, 181, par. 24: "Contra: Si consequens plus repugnat alicui quam antecedens, ergo magis impossibile est consequens esse verum cum illo quam antecedens. Sed ad antecedens esse verum cum aliquo sequitur consequens esse verum cum illo. Ergo ex minus impossibili sequeretur magis impossibile, et hoc per se et formaliter. Sed hoc est impossibile, quia minus impossibile respectu magis impossibilis habet rationem possibilis. Sed ex possibili numquam sequitur impossibile formaliter et per se. <Ergo ex minus impossibili numquam sequitur magis impossibile formaliter et per se>."

45 BURLEY 1972. This is the last commentary that Burley wrote on Aristotle's *Physics*, and was written around 1324–1337; cf. ASHWORTH 2013, 136.

diction in virtue of its terms" (*omnis propositio quae non includit contradictionem formaliter ex terminis*).⁴⁶ Interestingly, in this treatise Burley also equates the notion of *believability* with that of *imaginability*, using the two as synonymous and saying that any proposition which is not syntactically contradictory is both believable and imaginable (*opinabilis et imaginabilis*). Burley uses this idea of *opinabilitas* to describe the sorts of propositions that are employed in mathematics or geometry which involve entities and facts that are impossible with respect to the way things actually are but, being noncontradictory, are nevertheless imaginable and conceivable. The natural impossibilities that Burley has in mind are, for instance, the fact that for any given quantity another bigger quantity could be provided, or that the movement of a point creates a line.

The distinction between *opinabile* and *inopinabile* impossibilities is not echoed in obligational treatises after Burley, and seems to be abandoned in favour of other divisions of the modal term – like the one between impossibility *per se* and *per accidens*, or that between *absolute* (or *simpliciter*) and qualified impossibility – which remain in vogue in late medieval theories of obligations like those of Paul of Venice, Tarteys, Wyclif, Peter of Mantua and Paul of Pergola, to mention just a few.⁴⁷ Nevertheless, the association between impossibility and epistemic notions like intelligibility, believability or imaginability does not remain without followers in the first half of the 14th century. In the final section of this article, I will briefly consider three 14th-century accounts of *positio* in which an analysis of (im)possibility is offered and which echo, in different ways, the distinction between intelligible and unintelligible im-

46 BURLEY 1972, III f.81 ra. The passage is quoted and commented on in THIJSSEN 1985, 73.

47 See ASHWORTH 2015, 233 for the idea of *per se* impossibility in Tarteys and Paul of Venice; see PAUL OF PERGOLA 1966, 31 for Paul of Pergola; see STROBINO 2009, e.g. 82–4, 145–7 for Peter of Mantua. Another distinction that is found in early 14th-century Oxonian logic is the one between *impossibilis per se* and *de facto* (cf. KILVINGTON 1990, 249).

possibility that was developed in *Tractatus Emmeranus*, and then revisited by Sherwood and Burley. Although no author will use their exact terminology, the idea of this distinction seems to always lie in the background. The authors that I will consider here are William of Ockham (c. 1287–1347) and the two so-called ‘Calculators’ Thomas Bradwardine (c. 1300–1349) and Roger Swineshead (d. 1365?).

5. (Im)possibility and *positio* in Ockham, Bradwardine and Swineshead

5.1 William of Ockham

Ockham discussed the relation between impossibility and intelligibility in the part of the *Summa* devoted to consequences and to the art of obligations. As one might know, he believed that modal terms should not be restricted to the traditional alethic ones that were acknowledged by Aristotle, but that rather “almost innumerable” modalities exist,⁴⁸ among which many that we would now call epistemic modes, such as the terms ‘intelligible’ (*intelligibilis*), ‘believable’ (*opinabile*) and ‘credible’ (*credibile*) – the latter two being defined as “that to which the intellect could assent.”⁴⁹ Although Ockham does not divide impossibility into the two categories of *opinabiles* and *inopinabiles* used by Sherwood and Burley, there is some evidence that he accepts some impossibilities as being intelligible and believable. For instance, in the *Summa* Ockham mentions a rule according to which it would be invalid to infer, from the fact that something is intelligible or believable (*opinabile*), that it is also possible, although he maintains that the converted inference (from possibility to opin-

48 OCKHAM 1974, 341.

49 *Ibid.*, 398. Ockham seems to treat *opinabile* and *credibile* as synonymous here, as he states that “accipiendo ‘credibile’ pro illo cui potest intellectus assentire sive evidenter sive non evidenter.”

ability) is always valid.⁵⁰ Elsewhere in the same work, Ockham states that *opinabile* can be said of some things whose existence is entirely impossible, such as a chimera, an infinite line, or the void, which are thus intelligible and believable or imaginable.⁵¹ It seems, then, that Ockham – in contrast to some of his Parisian contemporaries like Buridan, for example – does not take the notions of intelligibility and believability as confined to the domain of possibility, but rather as intersecting that of impossibility as well.

Another piece of evidence relevant to reconstructing Ockham's view on the relation between impossibility and intelligibility is his discussion of *positio impossibilis*, which is also included in the theory of consequences advanced in the *Summa*, and was written around 1324.⁵² Here, Ockham states that in the species of obligation known as *positio*, both possible and impossible propositions are admissible as *posita* and could be conceded for the duration of the discussion.⁵³ The notion of impossibility (and that of its modal counterpart, necessity) is not treated as a 'monolithic' notion, though, but is further analyzed and divided into its different species. First, Ockham distinguishes between what has always been impossible (or necessary), and thus was so even before the beginning of the disputation, from what 'becomes' impossible or necessary *infra tempus obligationis*, that is, once the disputation has started

50 *Ibid.*, 641: "Tertio notandum quod omnis propositio in qua ponitur aliquis modus qui non potest competere nisi propositioni verae, infert illam de possibili; sicut sequitur 'omnem hominem esse animal est scitum, igitur omnem hominem esse animal est possibile'. Sed e converso non sequitur, nisi aliquando gratia materiae. Si autem talis modus possit competere propositioni falsae, tunc non infert illam de possibili, quamvis aliquando sequatur e converso. Unde non sequitur 'intellectum non esse animam intellectivam est opinabile, igitur intellectum non esse animam intellectivam est possibile'; sed e converso bene sequitur, quia omne possibile est opinabile."

51 *Ibid.*, 366.

52 For a discussion of this part of Ockham's text, see STUMP 1989, 262 ff.; YRJÖNSUURI 2000, 65–6; GELBER 2004, 187–8.

53 As Ockham points out, this is one of the differences that distinguish *positio* from *casus*, since the latter can only be used to discuss a possible – but counterfactual – situation.

and in consequence to something that has been put forward within it. This distinction resembles the one between *per se* (or absolute) and *per accidens* impossibilities that was common in the 13th century. Whereas impossibilities *per accidens* may be admitted and conceded in any disputation, even those that take their move from a possible *positum*,⁵⁴ Ockham thinks that propositions that are *per se* and sempiternally impossible are admissible only in the species of *positio* that starts with an impossible *positum*. However, not any impossibility is admissible, and Ockham further distinguishes between two categories of *per se* impossibilities: those that are explicitly contradictory or evidently entail contradictions, and do so with respect to any possible interpretation (“*illa propositio impossibilis quae manifeste apud omnem intellectum infert contradictoria*”), and those that are not patently contradictory, nor entail contradictions by means of self-evident inferential rules. While the latter are admissible as *posita*, the former are not. Ockham provides numerous examples of admissible *per se* impossibilities, including pragmatic, natural or doctrinal impossibilities: it is admissible, for instance, that: (i) you respond in *positio* conceding that you are dead; (ii) a man is capable of braying; (iii) a man is not capable of laughter; (iv) God does not exist; (v) God is not three persons; (vi) the Holy Spirit does not proceed from the Son; (vii) God is not wise. Unfortunately, we are not provided with any examples of self-evident and inadmissible impossibilities, but what Ockham probably has in mind are the impossibilities that Sherwood and Burley also called *inopinabiles*, such as ‘a man is not a man’, uttered in a situation in which men exist.

Notice that both admissible and inadmissible impossibilities – ‘a man is capable of braying’ or ‘God does not exist’ on the one hand, and ‘a man is not

⁵⁴ When an impossibility or necessity of this sort is conceded in a disputation of *positio possibilis*, however, the respondent must make sure to react to this proposition (either granting or denying it) consistently throughout the entire disputation, in line with the first reaction that he provided to them.

a man' on the other – all include some sort of contradiction in them or lead to contradiction by means of valid consequences, as Ockham explicitly remarks.⁵⁵ Thus it is not their contradictory nature that discriminates admissible from inadmissible impossibilities, but rather the evident vs implicit quality of the impossibility involved. What Ockham seems to have in mind is thus the same demarcation used in earlier discussions on *positio impossibilis* between impossibilities that are intelligible or believable (for example those impossibilities that are indeed actually believed by “the infidels,” such as that God exists but is not *trinus et unus*)⁵⁶ and impossibilities that are utterly absurd and could not be entertained as an object of thought or belief by any rational interlocutor.

As in the case of earlier discussions of *positio impossibilis*, Ockham too believes that, in order to reason with impossible *posita*, a number of constraints should be placed on the inferential principles that we admit. Just like his predecessors, Ockham denies the validity of principles like *ex impossibili quodlibet*⁵⁷ and provides an epistemic definition of consequence according to which only *consequentiae* that are universally self-evident are to be accepted in such *positiones*. Therefore, even consequences like ‘if the Holy Spirit does not proceed from the Son, the Holy Spirit is not distinct from the Son’, which are

55 OCKHAM 1974, 741: “Ex istis patet quod multae propositiones includentes contradictionem, hoc est inferentes contradictoria, possunt poni positione impossibili, nec propter hoc sunt contradictoria concedenda, quia facta positione tali non omne sequens exposito est concedendum, sed multa sequentia sunt neganda vel non concedenda. Omnia enim quae non sequuntur evidenter, ita quod consequentia talis non potest fieri evidens ex naturalibus, non sunt concedenda propter positum; et hoc sive positum sit una propositio categorica sive sit copulativa ex multis categoricis.”

56 *Ibid.*, 740–1: “Unde multae consequentiae bonae sunt et multae condicionales verae, quamvis non sint evidentes nobis. Similiter, si quaeratur ‘an si Deus sit, Deus sit trinus et unus’, respondendum est quod sic, quamvis infidelis errans aliter responderet, quia illa condicionalis vera est, quamvis non sit evidens.”

57 More generally, any consequence that is material or *ut nunc* is inadmissible when dealing with impossible propositions.

formally and simply valid for Ockham, are inadmissible in the context of *positio impossibilis*, as are all those consequences in which the relation of following is not self-evident.⁵⁸ Similarly, Ockham denies the validity of consequences in which from a certain affirmation the negation of a natural opposite is inferred, such as ‘if something is a man, it is not a donkey’.⁵⁹

We thus have a number of elements in Ockham’s analysis of impossible *positio* that seem to take their origin from the textual tradition explored in the previous sections: the connection between impossibility and believability; the distinction between two kinds of *per se* impossibilities, which are divided on the basis of an epistemic criterion (being believable or manifestly unbelievable); and the correspondent epistemic characterization of the *consequentiae* that are applicable when reasoning from an impossible premise.

It has been noticed by some scholars that the use of *positio impossibilis* came to a decline after Ockham,⁶⁰ and indeed we do not find any analysis specifically devoted to this subject in later literature on obligations, which nevertheless continues to flourish in the 14th century. This does not mean, however, that the notion of impossibility plays no role in 14th-century accounts of *positio*. Quite to the contrary, reflections on the practice of *positio* continued to stimulate a debate on the nature and categories of impossibility. Two interesting cases are those of Roger Swineshead and Thomas Bradwardine, to whom I would like to turn in conclusion of this survey.

58 See e.g. OCKHAM 1974, 741: “Omnia enim quae non sequuntur evidenter, ita quod consequentia talis non potest fieri evidens ex naturalibus, non sunt concedenda propter positum; et hoc sive positum sit una propositio categorica sive sit copulativa ex multis categoricis.”

59 Ockham does not give this example, but he speaks of the invalidity of consequences *de negatione repugnanti*, which makes me think that he has the *locus ab oppositis* in mind: “Omnia enim quae non sequuntur evidenter, ita quod consequentia talis non potest fieri evidens ex naturalibus, non sunt concedenda propter positum; et hoc sive positum sit una propositio categorica sive sit copulativa ex multis categoricis.”

60 Cf. SPADE 1982, 5; MARTIN 1992, 126; GELBER 2004, 189.

5.2 Roger Swineshead

Roger Swineshead's tract on obligations, written around 1330–1335, marks a significant departure from the previous rules of *ars obligatoria*, initiating what was called the 'new way' of answering to obligational disputes, as opposed to the 'old way', represented by Burley.⁶¹ As some scholars have remarked, Swineshead did not admit impossible propositions as *posita* in obligational disputes, limiting himself to the domain of possible (though usually false) postulations.⁶² This is clear when he lists the twelve *suppositiones* at the basis of *ars obligatoria*. Here, he maintains that a proposition whose truth value is not subject to change beyond the limits of the disputation may not be posited. What he means is that all propositions that are necessary or impossible *per se* – that is to say, sempiternally and invariably so – are inadmissible as *posita*. As Ashworth has shown, this seems to have been a widely shared view in the literature on obligations in the period of the Calculators, and especially in the second half of the 14th century, when several authors explicitly restrained *posita* to propositions that are not *per se* impossible or impossible *simpliciter*.⁶³ A man being a donkey was the standard example for this sort of impossibility.

This does not exclude, though, the admissibility of other kinds of impossible statements, namely, those that 'become' impossible by virtue of a certain fact happening in the course of time (*per accidens* impossibilities) or as a result of something that has occurred in the dispute itself, such as the phrase 'nothing has been posited to you' (which we could call, again follow-

61 Swineshead's *Obligationes* are edited in SPADE 1977. See Spade's introduction to this edition for a discussion of the dating, influence, and content of Swineshead's work.

62 Cf SPADE 1977, 254 n. 10; see also DUTILH NOVAES 2006, 129.

63 See for instance Tarteys and Paul of Venice, quoted in ASHWORTH 2015, 233. Peter of Mantua denies that propositions that are impossible *simpliciter* could be admitted and conceded (see ed. STROBINO 2009 81, 26–9; 82, 46–7).

ing Ashworth,⁶⁴ pragmatic impossibilities). So there are at least *some* impossibilities that can be used as *posita* according to Swineshead and his contemporaries. And in fact, the impossibilities that Swineshead considers inadmissible can be restricted further to only those *per se* impossibilities that would entail, by virtue of a valid consequence, something that is ‘more improbable’ (*improbabilius*) and ‘more absurd’ (*maius inconveniens*) than they are – in other words, impossibilities whose admission would lead to something more impossible and thus to an explicit contradiction.

Swineshead’s use of the distinction between the probability and improbability of propositions is worth briefly considering at this point. It is a distinction that is put forward at the beginning of his tract on obligations, where Swineshead says that every proposition is either probable or improbable, and that both categories may be admitted in *positio*. These two kinds of propositions are then distinguished into those that are *simpliciter* and *non simpliciter* (im)probable. A proposition is probable *simpliciter* if it is correctly demonstrable, and improbable *simpliciter* if it is contradictory (*repugnans*) to something that can be or has been demonstrated. Propositions are probable *non simpliciter* if we have reasons to hold them as true and believe that they can be demonstrated – reasons that can be either well-grounded or simply apparent; and propositions are *non simpliciter* improbable if we have reasons to believe that their opposite can be demonstrated and should be held as true.⁶⁵ Now, it becomes clear from what Swineshead says next that propositions which are improbable *simpliciter* (i.e., incompatible with evident and demonstrable truths) cannot be held as *posita* in an obligation, whereas prob-

64 ASHWORTH 2015, 234.

65 Notice that the same proposition, if it is neither evidently true nor evidently false, can be both probable and improbable *non simpliciter*, i.e., there may be reasons to justify both its truth and its falsity. ‘The Sun is bigger than the Earth’ is one such propositions, Swineshead says.

abilities and improbabilities *non simpliciter* ('the Sun is not bigger than the Earth' being the example provided) are admissible in *positio*, so the respondent can be obligated to concede them. Improbable propositions are only admissible insofar as they do not entail anything more improbable: this is because a good consequence, according to Swineshead, should always lead from what is more improbable to what is less improbable, and never the other way around. Swineshead also rephrases this rule by saying that one is never entitled to infer something which is 'more absurd' (*majus inconueniens*) from something 'less absurd' (*minor inconueniens*).

The terminology and the principles invoked by Swineshead at this point have not been fully understood by modern commentators. Yrjönsuuri proposed that, when advancing the aforementioned principle from *majus* to *minus inconueniens*, Swineshead is putting forward an innovative rule, which has no precedents in the earlier literature on obligations.⁶⁶ In my view, however, what Swineshead says at this point on *probabiles* vs *improbabiles* propositions, and on the inferential rules governing them, actually echoes what earlier obligational treatises said on *opinabiles* and *inopinabiles* propositions and on the logical relationship between the two. Swineshead's terminology may very well be a reformulation of that very same distinction, since – as was already mentioned in Section 3 – the term '(in)*opinabilis*' was often used as synonymous with '(in)*probabilis*' in the context of medieval dialectic, to indicate a proposition that is fit to be held as an opinion or entertained as a belief, without being evidently or uncontroversially true (false). As was said in Sections 3 and 4, both Sherwood and Burley admitted that impossible propositions could be held as *posita* (even *per se* impossibilities, like 'a man is a donkey') – but only insofar as such impossibilities did not entail something entirely *inop-*

66 YRJÖNSUURI 2001(2), 21.

inabile, that is, something unbelievable or evidently false, like a patent contradiction. Although reshaping this matter substantially, Swineshead seems to appeal to the same intuition when saying that (some) impossibilities are inadmissible because they would entail something more improbable or improbable *simpliciter* (that is, incompatible with an evident or demonstrable truth). Sherwood and Burley also agreed that the believability of a proposition comes in degrees: one proposition may be more believable than others. Burley associated the different degrees of (un)believability with degrees of impossibility, saying that one should never allow for something that is unbelievable to follow from something believable, or for something that is more impossible to follow what is less impossible, whereas the converse entailment (from more to less impossible) is valid. All this strikingly resembles Swineshead's view that the more improbable cannot follow from the less improbable (whereas the converted entailment is valid), and that something which is more *inconueniens* cannot be conceded on account of what is less *inconueniens*. Rather than introducing an innovative principle in his theory, what Swineshead seems to be doing is providing new terminology and a new systematization to concepts and rules that were developed in the context of *positio impossibilis*.

5.3 Thomas Bradwardine

Gelber argues that the use of impossible *positio* disappeared after Ockham not because of a lack of interest in the postulation of impossibilities, but rather because "impossible *positio* was subordinate to some greater frame of possibility," so that the discussion about the notion of possibility ultimately "swallowed up the impossible."⁶⁷ Even though Gelber advanced this interpretation

⁶⁷ GELBER 2004, 189.

thinking of Robert Holcot's use of *positio*, I believe that her suggestion fits well with the interpretation of another use of *positio* in the first half of the 14th century: the one advanced by Thomas Bradwardine in *De causa Dei* (1344).⁶⁸ Bradwardine's use of *positio* in this work is limited to what he calls *positio possibilis*, so he seems to exclude the use of impossible propositions as initial postulations in a dispute. But his notion of possibility is formulated in such a way that some of the impossibilities that Sherwood, Burley and Ockham considered as intelligible or believable – and as such, as positable – are now subsumed under the category of absolute or *per se* possibility, as Bradwardine labels it. According to his definition of 'possible' – which Bradwardine describes as the usual understanding of the word at the time – any statement is possible which does not include, formally and by virtue of itself (*per se*), an unqualified contradiction. Any statement which satisfies this criterion can be posited and conceded "*pro possibili*" in a *positio* disputation. The admission of any such possibility, Bradwardine points out, never entails a formal contradiction, provided that the inferential principles that are used are "good and formal consequences."⁶⁹ Possibility in this sense is contrasted with what Bradwardine calls absolute or *per se* impossibility, which applies to what *per se* and formally includes a contradiction. Including a contradiction is the same as in-

68 BRADWARDINE 1964. For Bradwardine's use of *positio*, see also MARTIN 1990.

69 BRADWARDINE 1964 I.1, 2: "Sumatur quoque possibile ad communem modum loquendi, vel si oporteat maxime absolute, pro illo videlicet quod per se et formaliter simpliciter contradictionem, seu repugnantiam non includit: Ex quo scilicet, posito et admissio pro possibili absolute secundum speciem obligationum, quae positio nominatur, nusquam in consequentia bona et formali simpliciter, sequitur impossibile absolute, quod scilicet per se et formaliter simpliciter contradictionem includit. Omnis namque repugnantia contradictionem importat et parit." Later in the text, Bradwardine states that this meaning of possibility is the one that is in use among logicians (*apud Logicos*): see *Ibid.* I.1, 4. The definition that Bradwardine gives here of what we may call a 'logical possibility' seems to clash, at least to the modern reader, with the temporal idea of possibility that Bradwardine uses later in the same passage of the text, where he claims that 'absolute possibility' is what *can be*, what *could have been* and what *will be able to be*. On Bradwardine's temporal account of the possible see also MARTIN 1990, 583 ff.

cluding some incompatibility (*repugnantia*), he goes on, because any incompatibility either entails or is equivalent to a contradiction.⁷⁰

The sense of possibility that Bradwardine proposes here is wide enough to include several entities, facts or statements that were earlier categorized as impossible – perhaps conceivable or imaginable, but nonetheless impossible. For instance, Bradwardine says that if we take ‘possible’ in the sense of *per se* and absolute possibility, entities like an infinite straight line or an infinitely rarefied medium are also possible insofar as they do not entail any contradiction, although they are *per naturam* impossible.⁷¹ Similarly possible, on such an account, are situations in which God creates something out of nothing and instantaneously.

With respect to his understanding of modalities, Bradwardine seems to go in the opposite direction from the one that was taken by the authors in the preceding sections, from the anonymous author of *Tractatus Emmeranus* to Burley and Ockham. During the 13th and the early 14th century, logicians made an effort to provide an analysis of impossibility more fine-grained than the one inherited from their predecessors, distinguishing between various senses and even various degrees of impossibility, and providing logical rules to model the behaviour of these different kinds of impossibility. The discussion of obligations, and particularly of *positio*, provided a good context for this analysis of impossibility (together with other contexts, such as the discussion of *syncategoremata*), and the distinction between conceivable and inconceivable impossibilities seems to have arisen in connection with this interest

⁷⁰ This is a point on which Bradwardine differs from what Ockham said in his account of *positio* included in the *Summa*: Ockham claimed that, even though any impossible proposition contains some sort of incompatibility (repugnancy), not all of them give rise to a formal contradiction. Bradwardine, on the contrary, thinks that any impossibility either entails or is equivalent to a contradiction.

⁷¹ BRADWARDINE 1964 I.1, 4.

in the nature of the impossible. Bradwardine's interest and aim in discussing *positio*, on the other hand, is not in the analysis of impossibility, but rather in subsuming some of the things that were earlier categorized as impossible under the wider category of absolute possibility. The latter, and not the former, is the central modal notion of Bradwardine's theory, whereas impossibility is defined and treated as ancillary to it.

Bradwardine has both metaphysical and theological reasons for giving such priority to possibility over its modal counterpart. As has been argued by other scholars, the argumentation that he offers in *De Causa Dei* is centrally based on a metaphysical foundation of modalities in God: Bradwardine thinks that necessity, possibility and impossibility all have their ultimate cause in God and thus depend on his existence and nature.⁷² What has not been highlighted, however, is that such a metaphysical and theological plan also requires a rigid hierarchical ordering among modalities, according to which the possible is (causally and conceptually) dependent on the necessary,⁷³ and the impossible on the possible. Impossibility is thus said to be both metaphysically and logically subordinate to the possible, and its definition entirely reducible to it.⁷⁴ Within this conceptual framework, it is not surpris-

72 On this view see in particular KNUUTTILA 2003 and FROST 2012.

73 'Pure necessity' is said to be the "*radix prima et fundamentum*" of the other modalities, cf. BRADWARDINE 1964, I, 13, 203.

74 *Ibid.*, I, 13, 203–208. Here, Bradwardine also says that the possible 'causes' the impossible. (*Ibid.* I, 13, 207) God, being the cause of necessity and of possibility, is thus the ultimate cause of impossibility as well. For an analysis of Bradwardine's idea that impossibilities are metaphysically founded in God, see FROST 2012, 372–3. See for instance these short extracts from chapter I of *De Causa Dei*: "Ex hoc potest cognosci quòd necessarium est prius impossibili: Possibile enim est prius impossibili sicut affirmatio negatione, habitus privatione, et esse non esse; et necessarium est prius possibili, sicut novissime probatur"; "Item impossibile et possibile dicuntur ad invicem relative secundum privationem [...]; sed de impossibili cum sit pure non ens, nulla relatio per se et essentialiter potest consurgere seu fundari, quia tunc duo pure non entia possent referri ad invicem per se et essentialiter, sine coexigentia alicuius existentis omnino: Ista ergo relatio per se et primo fundatur in extremo positivo, scilicet in possibili, et emanet ab eo, et sic attribuitur, et accidit quodammodo extremo alteri privativo, sicut de relativis tertij

ing that we find no mention of impossible *positio* in Bradwardine. Possibility indeed “swallowed up the impossible,” to quote Gelber once again.

6. Conclusion

The distinction between *conceivable* and *inconceivable* impossibilities, which emerges in the 13th century as part of the general re-analysis of the nature and kinds of impossibility, is closely connected to the development of *ars obligatoria* and the use of *positio*. While discussing the subspecies of obligation called *positio impossibilis*, several authors claimed that an impossible premise could be posited and conceded at the beginning of a dispute at the condition that: (i) this impossibility does not entail anything ‘more impossible’ or outright contradictory; and that (ii) the impossibility involved is an ‘intelligible impossibility’: one that is fit to be held as an opinion or entertained as a belief by a rational interlocutor. A debate then arose on two main issues: which impossibilities count as intelligible (and particularly, whether natural impossibilities like ‘a man is a donkey’ or ‘a man is not an animal’ are among these), and which sort of *inferentiae* are admissible when reasoning from statements of this sort.

The most interesting feature emerging from this debate is that the standard criterion for the validity of natural consequences, which required a metaphysical or semantic connection between antecedent and consequent, was called into question. Authors who accepted impossible *posita* also thought that nature cannot serve as the proper *vis inferentiae* when reasoning

modi proximo dicebatur. Possibile ergo causat istam relationem et impossibilitatem, et necesse est prius possibili esse, et cause illius, sicut superius est ostensum, et Deus est prima causa omnium possibilium et necessariorum, ac omnium causationum illorum”; “Possibile ergo est prima causa huius repugnantiae et impossibilitates, et Deus est prima causa omnis alterius causationis et causae.”

from the impossible, and rather turned to epistemic, doxastic or psychological principles to define the relation of following. The author of *Tractatus Emmeranus* contrasted consequences that are valid *respectu naturae* with those that are valid *quantum ad intellectum*, thinking that only the latter are applicable to intelligible impossibilities; Sherwood thought that positing an impossibility as *opinabile* would ‘destroy’ the source of validity of a natural consequence; both Burley and Ockham restrain *consequentiae naturales* only to those the truth of which is indubitable and manifest to every rational agent. All these analyses of *positio impossibilis* thus stand in contrast with the naturalistic account of consequence and of the relation of (in)separability between things that a consequence is supposed to capture. In the first half of the 14th century, for either logical or theological reasons, the main interest of authors discussing *positio* shifted from the notion of impossibility to that of possibility. Nevertheless, some ideas that were developed by earlier authors about the connection between impossibility and intelligibility survive in a revised form, as both Swineshead and Bradwardine’s uses of *positio* demonstrate.

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JOHN DUMBLETON ON INSOLUBLES: AN EDITION OF AN EPITOME OF HIS SOLUTION TO INSOLUBLES

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Abstract: This paper provides a philosophical analysis and a new edition of an anonymous *Epitome (Compendium)* of John Dumbleton's solution to the semantic paradoxes (*insolubilia*). The first part of this paper briefly presents Dumbleton's cassationist solution to the semantic paradoxes, which the English philosopher proposes in his *Summa Logicae*, written in the 1330s–40s. The second part investigates the solution to various types of insolubles proposed by the anonymous author of the *Epitome*. The third part provides a new critical edition of the Latin text – a first edition was edited by Bottin in 1978 – and an English translation.

Keywords: Semantic paradoxes; Cassationism; John Dumbleton; 14th-century philosophy; Oxford logic.

1. Introduction

The manuscript 397 Scaff. XVIII of the Biblioteca Antoniana in Padua ends with an anonymous *Epitome (Compendium)* of John Dumbleton's solution to the semantic paradoxes (*insolubilia*). Bottin edited this *Epitome of Dumbleton's Insolubles* more than forty years ago, when knowledge of the *insolubilia*-literature was still quite limited.¹ Since then, much research has been done and many primary sources have been studied and edited, including Dumbleton's *Summa Logice*, which is the source for the *Epitome*. In preparing the edition of the *Insolubles* from Dumbleton's *Summa Logice* recently, we realized that a

* The present work was funded by Leverhulme Trust Research Project Grant RPG-2016-333: *Theories of Paradox in Fourteenth-Century Logic: Edition and Translation of Key Texts*.

1 BOTTIN 1978. The description of the manuscript and its content can be found in ABATE, LUISETTO 1975, 332–333.

new edition of the short *Epitome* was needed, in which on the one hand, some corruptions in Bottin's version could be corrected, and which, on the other hand, could serve to elucidate the *Epitome's* content and provide some context by outlining Dumbleton's own solution. Accordingly, in the first part of this paper, we briefly present Dumbleton's cassationist solution to the semantic paradoxes. In the second part, we analyse the solution to various types of insolubles proposed by the anonymous author of the *Epitome*. In the third part, we provide a critical edition of the Latin text and an English translation.

1. John Dumbleton's semantics and paradoxical propositions

Little is known about the life of John Dumbleton. From the end of the 1330s Dumbleton was active in Oxford, where he was part of the group of Oxford Calculators; he seems to have spent a triennium in Paris (1344–7) for studying theology, then returned to Oxford and probably succumbed to the Black Death after 1348. In Oxford Dumbleton wrote his only known work, the *Summa Logice et Philosophie Naturalis* (hereafter abbreviated as *SLPN*), which is preserved in 21 manuscripts and is mostly unpublished. The *SLPN* is a massive work covering logic (Part 1) and natural philosophy (Parts 2–9) and seems to be incomplete. Indeed in several passages Dumbleton refers to a tenth Part concerning universals which is not found in any of the 21 manuscripts and plausibly was never written.² The *Summa Logice* is the first Part of the *SLPN*, is contained in 19 of the 21 manuscripts preserving the *SLPN* and is subdivided into three main parts dealing with various semantical, logical and epistemological topics. For our purposes only the first part is relevant, where

² On Dumbleton and the *Summa Logice et Philosophie Naturalis* see WEISHEIPL 1969(1) and 1969(2); SYLLA 1991; SYLLA 2011.

Dumbleton offers his account of linguistic meaning and his solution to the semantic paradoxes.³

For Dumbleton, categorematic spoken and written terms have a conventional signification acquired through a first act of bestowing a name on a thing (*impositio*) after which a relation between a term's intention (*intentio termini*) and a thing's intention (*intentio rei*) is established in the subject's mind. Signification always involves active participation of a subject and in his account of linguistic meaning, Dumbleton describes a term's signification as a mental process triggered in the reader's or hearer's mind by a term. Spoken and written terms have their proper intentions, namely specific physical features such as shape for written marks or frequency for sounds, which the subject perceives while receiving an external stimulus. When the subject perceives a term's intention, she retrieves from her mind/memory the thing's intention previously associated with that specific term's intention through the *impositio*: "for a term to signify something in a simple way is to actualize, i.e., call to mind that thing's intention by means of the term's proper intention, <and> that thing is said to be signified by that term in normal usage."⁴ Thus the meaning of terms are intentions, *viz* mental representations, of things (*intentiones rerum*). Unlike terms, intentions naturally signify their significates, be they simple objects or complex objects like propositions, and similarly to terms, intentions signify only when the subject entertains them – "while there is actual apprehension through them."⁵

Dumbleton's notion of signification as involving subjective activity also applies to propositions. A well-formed spoken or written string of words is a

3 An analysis of Dumbleton's semantics and solution to the paradoxes is found in READ FORTHCOMING.

4 DUMBLETON IN PREPARATION, § 2.1.1.

5 DUMBLETON IN PREPARATION, § 13.1–13.2.

proposition properly speaking only if there is a person reading, uttering or hearing it, while a well-formed mental complex is a proposition only insofar as there is some mind forming and entertaining it: “A proposition only exists externally and in the mind while there is actual composition through the ⟨rational⟩ soul.”⁶ Since the signification of a proposition depends on the signification of its parts, if at least one of its parts is meaningless, the whole proposition fails to express a complete meaning and to be truth-apt and so is not a proposition properly speaking. This can happen in the case of sentences with at least one context-dependent term, e.g. demonstratives, whose meaning is left undetermined in the context, or it can happen with sentences containing expressions signifying complex things, like ‘truth/true’ or ‘proposition’. And therein lies the problem with semantic paradoxes, or insolubles, like the Liar.

Dumbleton’s diagnosis and solution to insoluble propositions is expressed using the language of obligations, which were regimented discussions commonly held by an opponent and a respondent in 14th-century logic classrooms. In short, in an obligational discussion an opponent proposes to a respondent a background context or scenario (*casus*) and a first proposition (*positum*), which is usually false in the given scenario. If the respondent admits the *positio*, the discussion starts and the opponent proposes further propositions which the respondent must grant, doubt or deny on the basis of the obligational rules without falling into contradiction.⁷ For Dumbleton, insolubles are propositions containing expressions signifying a propositional complex which become problematic within specific contexts involving direct or indirect self-reference: “An insoluble is a proposition which is inferred to be

6 DUMBLETON IN PREPARATION, § 13.3.

7 On obligations see DUTILH NOVAES, UCKELMAN 2016.

true and false when an apparently possible scenario is admitted.”⁸

In the *Summa Logice* Dumbleton considers six scenarios (*casus*) generating insolubles. The first is the scenario of a version of the Liar paradox in which there is only one proposition, like ‘A falsehood exists’ – but that same scenario, says Dumbleton, also generates paradoxes like ‘A truth exists’, ‘A proposition exists’ or ‘No falsehood exists’. In Dumbleton’s semantics, the term ‘falsehood’ always signifies a proposition other than that of which it is part; to get the complete meaning of, and thus to understand ‘A falsehood exists’ the term ‘falsehood’ must be replaced by the proposition it stands for. However, in the proposed scenario this is the only proposition and so cannot refer to a second proposition, hence it fails to convey a full meaning and is not truth-apt and, consequently, cannot be considered a proposition properly speaking. It is clear that a scenario like that is impossible, therefore should not be admitted⁹. Dumbleton adopts the same approach with the fourth and fifth scenarios he considers. In the fourth scenario, there is only one Socrates who only says proposition A: ‘Socrates is a liar’ – or alternatively ‘Socrates is an oath-breaker’, ‘I am a liar’, ‘I say nothing’. Terms like ‘oath-breaker’ or ‘liar’ signify a propositional complex which should exist before Socrates utters A – or a proposition similar to it; since “what is naturally posterior does not actually exist without what is prior, so Socrates cannot comprehend himself to be a liar unless he has in mind a proposition different from that.”¹⁰ But A is the only existing proposition, therefore the scenario is impossible and should be rejected. Similarly impossible is the fifth scenario, where the only existing proposition is the one believed by Socrates, namely ‘Socrates is deceived’, which requires the prior existence of a proposition about which So-

8 DUMBLETON IN PREPARATION, § 18.0.

9 DUMBLETON IN PREPARATION, § 18.1–18.1.2.3.

10 DUMBLETON IN PREPARATION, § ad 21.1.

crates was deceived.¹¹

His solution to insolubles of the third and sixth scenarios relies on the subordination of extramental language to mental language. In the third scenario, there is the classical self-referring Liar proposition 'This proposition is false'; Dumbleton says that if this is an extramental proposition, then the scenario should be doubted as incomprehensible "because it is not specified what precisely should be comprehended through the term 'this'."¹² But if it is mental, then the scenario should be rejected because Socrates can form the thought 'This proposition is false' only if he already has its subject in mind; but if he had it, then he would have 'This proposition is false' in his mind before he formed the thought 'This proposition is false', which is impossible.¹³ In the sixth scenario, Socrates only says A: 'Socrates says a falsehood'; Dumbleton accepts the scenario since it does not rule out the possibility that 'falsehood' refers to a mental proposition. Thus if A refers to a true mental proposition, A is false, and conversely A is true if it refers to a false mental proposition.

The second scenario considered by Dumbleton is the less problematic, provided it is correctly understood. It includes three propositions: the two true propositions A: 'God exists' and B: 'A man exists', and a mental proposition C: 'Every truth is one of these' – or alternatively 'These are all the truths' – referring to A and B. For Dumbleton the scenario should be admitted insofar as it is not intrinsically impossible and allows the respondent to establish the truth-value of C. Indeed since self-reference is banned and C's signification is restricted to A and B, then C is true in that scenario, as can be seen by making its meaning explicit by replacing 'truth' with its significates, namely A

11 DUMBLETON IN PREPARATION, § 21.2–21.3.

12 DUMBLETON IN PREPARATION, § ad 18.3.

13 DUMBLETON IN PREPARATION, § 18.5.

and B, thus obtaining 'Each of "God exists" and "A man exists" is one of these'.¹⁴

For Dumbleton the problem with semantic paradoxes is that they i) are, directly or indirectly, self-referential, where the type of 'reference' meant here is signification; ii) contain terms signifying complex things. Dumbleton's solution, which is a form of *cassationism*,¹⁵ consists in showing that, at least in his semantics, the scenarios generating insolubles are impossible and that within such scenarios insoluble propositions fail to express a complete meaning, are not truth-apt and cannot be considered as propositions properly speaking.

2. The *Epitome* of Dumbleton's solution to insolubles

Dumbleton's solution seems to have enjoyed some circulation both in and outside Oxford, as testified by the fact that it is listed among famous opinions in some 14th-century treatises on insolubles. Further evidence of its dissemination is to be found at the end of the manuscript 397 Scaff. XVIII of the Biblioteca Antoniana in Padua. At folios 118v-119v there is an *Epitome* (*Compendium*) of Dumbleton's view on semantic paradoxes that the explicit labels 'Epitome of Dumbleton's insolubles according to the Oxford usage (*secundum usum Oxonie*)'. The specification 'Oxford usage' suggests that this short *Epitome* was used as (perhaps part of) a textbook in Oxford, plausibly in the second half of the 14th century. As remarked by Bottin (1978), the content and arrangement of the *Epitome* reveal its didactic purpose; indeed the text starts abruptly listing six groups of insoluble propositions or scenarios (see *infra* §§ 1-1.6), then offers concise examples for each group (§§ 2-2.6) and fi-

14 DUMBLETON IN PREPARATION, § 18.2.2-ad 18.2.5

15 On Cassationism see SPADE 1987.

nally the diagnosis and solution to each insoluble scenario, regarding two of which some objections are raised and solved (§§ 3–ad3.6). No mention is made of Dumbleton’s semantics which, as seen, underpins his diagnosis and cassationist solution to semantic paradoxes. Actually, what we find in the *Epitome* is not only a shortened and simplified version of Dumbleton’s original analysis of the six scenarios, but rather a revision of it. The most patent and evident difference is the different order in which Dumbleton and the *Epitome* analyse the various scenarios, as the following table shows:

Dumbleton	Epitome
<p><i>1st scenario</i></p> <p>There is only one proposition, namely: A truth exists (or: A falsehood exists, No falsehood exists, A proposition exists).</p> <p>Solution</p> <p>The scenario is rejected as impossible for “necessarily some proposition is required to be the subject” of the insoluble, which however is the only existing proposition.</p>	<p><i>1st scenario</i></p> <p>There is only one proposition, namely: A truth exists (or: A falsehood exists, No falsehood exists, A proposition exists).</p> <p>Solution</p> <p>The scenario is rejected as impossible since the subject or predicate of the insoluble proposition supposits for a proposition, but the insoluble is the only proposition.</p>
<p><i>2nd scenario</i></p> <p>There are three propositions, namely A= God exists, B= Some man exists, C= Every truth is one of these (or: These are all the truths), referring to A and B.</p> <p>Solution</p> <p>The scenario is accepted and C is true “since the subject of C is A or B, or both (conjunctively or disjunctively); and so C is true, signifying only like this: ‘Each of “God exists”</p>	<p><i>4th scenario</i></p> <p>There are three propositions, namely A= God exists, B= A man is an animal, C= Every truth is one of these (or: These are all the truths), referring to A and B.</p> <p>Solution</p> <p>The scenario is accepted and C is true.</p>

<p>and "A man exists" is one of these'."</p>	
<p><i>3rd scenario</i></p> <p>There is only one proposition, namely A= This is false (or: This is true), with 'This' referring to A.</p> <p>Solutions</p> <p>i) A is extramental: the scenario is doubted insofar as it is incomprehensible since the referent is not specified</p> <p>ii) A is mental: the scenario is rejected as impossible because "Socrates would have A in his mind before A existed."</p>	<p><i>2nd scenario</i></p> <p>There is only one proposition, namely A= This is false (or: This is not true), and 'This' refers to A.</p> <p>Solution</p> <p>The scenario is rejected as impossible since A would exist before it existed.</p>
	<p><i>3rd scenario</i></p> <p>There are three propositions, namely A= God exists, B= Each of these is true (referring to A and C), C= Not all of these are true (referring to A and B).</p> <p>Solution</p> <p>The scenario is rejected as impossible since "it follows that a proposition [<i>viz</i> B and C] would exist before it existed."</p>
<p><i>4th scenario</i></p> <p>Socrates utters only one proposition, namely: Socrates is a liar (or: Socrates is an oath-breaker)</p> <p>Solution</p> <p>The scenario is rejected as impossible since a mental proposition should correspond to the predicate 'liar' and "Socrates cannot comprehend himself to be a liar unless he</p>	<p><i>5th scenario</i> (not directly discussed in the <i>Epitome</i>)</p> <p>Socrates only utters the proposition: Socrates is a liar</p> <p>Solution</p> <p>The scenario is admitted and the proposition is false because the predicate 'liar' supposes for a proposition, but the only proposition is 'Socrates is a liar', which cannot refer to itself.</p>

has in mind a proposition different from that."	
<p>5th scenario</p> <p>Socrates believes only one proposition, namely: Socrates is deceived.</p> <p>Solution</p> <p>The scenario is rejected as impossible since "Socrates only comprehends that he is deceived if he actually has a proposition other than this one <in mind>."</p>	<p>6th scenario</p> <p>Socrates believes only one proposition, namely: Socrates is deceived.</p> <p>Solution</p> <p>The scenario is rejected as impossible.</p>
<p>6th scenario</p> <p>Socrates only utters A= Socrates says a falsehood (or: Socrates says a truth)</p> <p>Solution</p> <p>The scenario is admitted "insofar as to the term 'falsehood' there corresponds a mental proposition such that, if it is false and uttered by Socrates, A is true, and if not, A is false."</p>	<p>5th scenario</p> <p>Socrates only utters A= Socrates says a falsehood</p> <p>Solution</p> <p>The scenario is admitted and A is false because the predicate 'falsehood' supposits for a proposition, but A is the only proposition and self-reference is banned, so A "signifies that Socrates says a proposition which he does not say."</p>

This table also shows more substantial differences between Dumbleton's genuine solution and the *Epitome*. Firstly, the *Epitome* considers a scenario absent in Dumbleton, namely the 3rd scenario (see the fourth entry in the table) in which there is a flip-flop back and forth between two insolubles each of which signifies the other.

A second major difference is found in Dumbleton's 3rd scenario and in the *Epitome's* 2nd scenario (see entry three in the table), which generates the classical Liar paradox. Here, the only existing proposition is the self-referen-

tial A: 'This is false'. Since the scenario does not specify whether A is a spoken, written or mental proposition, Dumbleton proposes two different solutions, one for the extramental level and the other for the mental level. If A is extramental, then the scenario should be doubted as incomprehensible, while if A is mental, the scenario is impossible and so should be rejected. While Dumbleton spends many words on this insoluble, the *Epitome* offers a much simpler treatment of it in § 3.2.2, where it is briefly said why the scenario should be rejected as impossible, avoiding any reference to the mental-extramental distinction.

A third and more substantial difference between Dumbleton and the *Epitome* is the treatment of the scenario in which Socrates only utters proposition A: 'Socrates says a falsehood', which corresponds to Dumbleton's 6th and the *Epitome's* 5th scenarios (see the last entry in the table). Dumbleton spends only a few words on this scenario, saying that it should be admitted since A is a spoken proposition to which there corresponds a mental proposition, call it B, on which A's truth-value depends: if B is true and uttered by Socrates, then A is false; conversely if B is false and uttered by Socrates, then A is true. The author of the *Epitome* seems to have found this *casus* worthy of discussion. Unlike Dumbleton, the *Epitome's* solution is not grounded on the extramental-mental distinction and consists in admitting the scenario and saying that A is false.

The substantial difference between the *Epitome's* and Dumbleton's treatment of insolubles arising from this scenario can be fully appreciated looking at the 4th scenario considered by Dumbleton (see the fifth entry in the table). Here the only existing Socrates utters only proposition C: 'Socrates is a liar'; for Dumbleton a mental proposition should correspond to the predicate 'liar' since "Socrates cannot comprehend himself to be a liar unless he has in mind

a proposition different from that"; therefore the scenario is impossible and should be rejected. The *Epitome* does not directly analyse insoluble C, which it lists in § 2.5 among the insolubles generated in the 5th scenario along with A 'Socrates says a falsehood'; to all insolubles arising in that scenario, e.g. A and C, the *Epitome* gives the same reply, namely the one we saw earlier in the case of A: the scenario is admitted and the insoluble is false because its signification is false, for it signifies that Socrates says a proposition which he does not say (§ 3.5.2). Or, in other words, the insoluble is false because it lacks a referent. This solution radically departs from Dumbleton and comes closer to the approach adopted by the so-called restrictivists, whose most prominent 14th-century exponents were Walter Burley, William of Ockham, Walter Segrave and Robert Holkot. The (moderate) restrictivists banned self-reference, saying that a part cannot supposit for the whole of which it is part in the presence of a privative term like 'falsehood'; consequently they claimed that in a scenario such as the 5th in the *Epitome* insolubles like A are false for they cannot refer to themselves and, being the only existing propositions, they lack a referent and are therefore false.¹⁶ This is exactly what the *Epitome* says while discussing A in the 5th scenario, at § ad 3.5.2. Since A is an insoluble proposition, one can infer from it that it is both true and false; for us what is relevant is the argument that concludes that A is true, which is found at § 3.5.2: "If it is granted that Socrates says a falsehood and A signifies precisely like that, therefore A signifies precisely as it is and consequently A is a truth. And if so, since Socrates says nothing except A, it follows that Socrates says a truth, therefore if Socrates says a falsehood, Socrates says a truth." Having stated that A is false, the anonymous author wants to show how to block the inference of § 3.5.2 moving from A's being false to A's being true, and the terminology and strategy he uses come very close to that of the restrictivists:

¹⁶ On restrictivism see SPADE, READ 2021, § 2.4.

To the argument [*sc.* § 3.5.2], when one argues: ‘Socrates says a falsehood and A signifies precisely that Socrates says a falsehood, therefore A is a truth,’ I deny the inference. And one may respond in this way to every insoluble of the fifth group by admitting the whole ⟨argument⟩ until we reach the argument just denied or one like it, which should be denied. The reason why this inference is not valid is this, that in proposition A the predicate is a term suppositing for a (propositional) complex and no such term can supposit for a proposition of which it is the subject or predicate, hence it is required that it supposits for some other proposition. If the predicate ‘falsehood’ in that proposition ‘Socrates says a falsehood’ supposits for a proposition other than ‘Socrates says a falsehood’, the proposition ‘Socrates says a falsehood’ is false, because it signifies that Socrates says a proposition which he does not say.¹⁷

Thus, the *Epitome* is not a mere simplified précis of Dumbleton’s theory of insolubles, as the explicit states, but it is an interesting didactical synopsis that offers an eclectic solution, mainly based on Dumbleton, to six different kinds of semantic paradoxes, possibly the six most discussed types of paradoxes when the *Epitome* was composed.

3. The Latin text and English translation of the *Epitome*

The Latin text has been prepared on the basis of the Padua manuscript. Where the text transmitted by the manuscript posed grammatical, doctrinal or logical problems, we amended it ourselves or following Bottin. The critical apparatus records all the variants of the manuscript and of Bottin’s edition. We have adopted the medieval manuscript spellings, including e.g., ‘e’ for ‘ae’, ‘Sortes’ for ‘Socrates’, but have adopted minimal modern punctuation as the meaning of the text requires. The section headings and the division into paragraphs are ours.

In translating the text, we have tried to stay as close as possible to the Latin text and to be as consistent as possible. In some cases, we have inserted

¹⁷ See *infra*, § ad 3.5.2.

words in ⟨angle brackets⟩ in order to make the translation more explicit and clearer; in a few cases, where the Latin text was not completely clear, we have opted for a free translation that reflects our understanding of the text.

Conspectus Signorum

In textu

⟨...⟩ verba ab editoribus addita includunt

[] uncis angulatis indicantur verba ab editoribus deleta

Conspectus Abbreviationum in apparatu critico

corr. = correximus

inv. = invertit, -erunt

ms. = codex

om. = omisit, omiserunt

Anonymous, Compendium solutionis insolubilium magistri Johannis de Dulminton secundum usum Oxonie.

Ms = Padua, Biblioteca Antoniana 397 Scaff. XVIII, ff. 118v–119v.

- 1.0 (f. 118v) Insolubilia sive insolubilium casus per sex ordines intendo distinguere in presenti et de quocumque insolubili casu proposito pro maiori¹⁸ parte promcius reddet suus ordo responsionem.
- 1.1 Primus ordo est quando supponitur aliqualem propositionem solam esse cuius subiectum vel predicatum supponit pro complexo.
- 1.2 Secundus ordo supponit aliquam propositionem per suum subiectum vel predicatum demonstrari.
- 1.3 Tertius ordo est quando supponitur aliquam propositionem esse cuius <subiectum vel>¹⁹ predicatum pro aliqua vel aliquibus propositionibus supponit quam prius esse oportebit naturaliter quam ipsius subiectum vel predicatum pro tali vel talibus suppositis <supponat>²⁰; nec etiam poterit illa nec ille pro quibus supponit talis propositionis²¹ subiectum vel predicatum esse naturaliter prius <quam>²² fuerit illa propositio cuius subiectum vel predicatum pro tali vel talibus supponit, ita quod breviter in omni casu tertii ordinis sequitur illam²³ esse antequam ipsam²⁴ esset.

18 maiori (corr. cum Bottin)] minori ms.

19 subiectum vel (corr.)] om. ms. Bottin

20 supponat (corr. cum Bottin)] om. ms.

21 propositionis (corr.)] propositio ms. Bottin

22 quam (corr. cum Bottin)] om. ms.

23 illam (corr.)] illud ms. Bottin

24 ipsam (corr.)] ipsum ms. Bottin

1.3.1 Verbi gratia, sit b ista propositio:

Deus est,

et sit a illa:

Quelibet illarum est vera,

demonstrando²⁵ b et c propositiones, et sit c illa propositio:

Non quelibet illarum est vera,

demonstrando²⁶ a et b propositiones. Tunc subiectum et predicatum in a supponunt²⁷ pro b et c propositionibus et ideo ad hoc quod a sit oportet quod prius fuerint²⁸ b et c propositiones pro quibus supponit. Sed c non potest²⁹ esse nisi prius fuerint a et b propositiones, cum³⁰ subiectum et predicatum supponunt pro a et b, et ita sequitur quod a esset antequam esset, et eodem modo sequitur de omni insolubili tertii ordinis.

1.4 Quartus ordo convenit cum tertio in toto, hoc excepto, quod non requiritur³¹ insolubile quarti ordinis prius <esse>³² quam esset propositio pro qua subiectum vel predicatum [supponit]³³ illius insolubilis³⁴ supponit.

1.5 Quintus ordo est quando supponitur aliquem hominem dicere, proferre, audire vel videre solum unam propositionem cuius subiectum vel predicatum supponit pro complexo, ut posito quod Sortes dicat illam propositionem et nullam aliam [tunc]³⁵:

Sortes dicit falsum,

25 demonstrando (*corr.*)] demonstrato *ms*, demonstrate *Bottin*

26 demonstrando (*corr.*)] demonstrato *ms*, demonstrate *Bottin*

27 supponunt (*corr.*)] supponit *ms*.

28 fuerint (*corr.*)] fuit *ms*, sint *Bottin*

29 sed c non potest] non possunt *ms*, *om.* *Bottin*

30 cum] tamen *Bottin*

31 requiritur] sequitur *Bottin*

32 esse (*corr.*)] *om.* *ms.* *Bottin*

33 predicatum (*corr. cum Bottin*)] predicatum supponit *ms*.

34 insolubilis (*corr. cum Bottin*)] insolubile *ms*.

35 aliam (*corr.*)] aliam tunc *ms.* *Bottin*

vel videat illam et nullam aliam:

Sortes videt falsum.

- 1.6 Sextus ordo <est>³⁶ quando supponitur aliquem credere solum unam propositionem cuius subiectum vel predicatum supponit pro complexo, ut posito quod Sortes credat illam propositionem et nullam aliam:

Sortes decipitur.

- 2.0 Istorum autem sex³⁷ ordinum non debet aliquis casus admitti nisi solum casus de quarto et de quinto ordine.

- 2.1 Primus ordo: verum est; falsum est; nullum verum est; nullum falsum est; propositio est; necessarium est; possibile est; impossibile est; omnis propositio universalis est falsa; nulla propositio universalis est falsa; tantum exclusiva falsa est; nulla est exceptiva falsa nisi ista, demonstrando se ipsam.

- 2.2 Secundus ordo: non est ita sicut illa significat; hec est falsa et hec significat aliter quam est; hec non est vera: per nullum tempus fuit; hoc verum contradictorium illius est verum; deus est et tantum prima pars istius copulative est vera; deus est et quelibet copulativa est falsa; homo est asinus et nulla copulativa est vera.

- 2.3 Tertius ordo: quelibet istarum est vera, demonstrato a, c; et non quelibet istarum est vera, demonstrato a, b; et quodlibet a est simile b; Sortes decipitur vel contradictorium illius disiunctive est verum; nullus deus est vel <illa>³⁸ disiunctiva est vera.

- 2.4 Quartus ordo: quodlibet verum est aliquod istorum; ista sunt omnia vera; deus est et homo est asinus; tantum unum istorum est verum;

36 est (*corr. cum Bottin*)] *om. ms.*

37 sex (*corr. cum Bottin*)] *sextus ms.*

38 illa (*corr.*)] *om. ms. Bottin*

deus est et tantum ista est vera; homo est et tantum ista est propositio.

2.5 Quintus ordo: Sortes dicit falsum; Sortes non dicit verum; Sortes est mendax; Sortes est periurus; Sortes profert falsum.

2.6 Sextus ordo: Sortes decipitur; Sortes non decipitur; aliquis homo decipitur; aliquis homo decipitur et Sortes est ille.

3.0 Ad omnia insolubilia primi ordinis eadem et consimilis est responsio et omnium illorum et consimilium est eadem probatio.

3.1 Verbi gratia, ponatur quod a sit illa propositio:

Falsum est,

sic significando precise et quod nulla sit nisi illa. Isto posito, vel a est verum vel falsum; si dicatur quod est verum, igitur ita est totaliter sicut illa significat et illa precise significat quod falsum est, igitur falsum est. Et cum nulla propositio sit nisi a, sequitur quod a est falsum, igitur si a est verum, sequitur quod a est falsum. Si dicatur quod a est falsum, tunc sic: a est falsum, igitur non est ita sicut illa significat et illa sic precise (f. 119r) significat, igitur <a est verum, igitur>³⁹ si a est falsum⁴⁰, sequitur quod a est verum⁴¹; et consimili modo arguitur de omni insolubili primi ordinis.

3.2 Pro solutione dicitur negando casum sive sit insolubile primi ordinis sive secundi⁴², sive tertii, diversa tamen causa est assignanda⁴³ pro impossibilitate casus in una quam in alia.

3.2.1 Causa quare casus de insolubili primi ordinis <est>⁴⁴ impossibilis est

39 a est verum, igitur (corr.)] *om. ms. Bottin*

40 falsum (corr.)] *verum ms. Bottin*

41 verum (corr.)] *falsum ms. Bottin*

42 secundi] *secundi ordinis Bottin*

43 assignanda] *assignando Bottin*

44 est (corr. cum Bottin)] *om. ms.*

quia propositio illa non potest esse nisi alia sit, cum ista convenire debeat subiecto vel predicato illius insolubilis, cum⁴⁵ subiectum vel predicatum propositionis est terminus supponens⁴⁶ pro complexo, sicut prius est argutum.

3.2.2 Causa quare <casus>⁴⁷ de insolubili secundi ordinis est impossibilis est hec, quia proponitur quod ipsamet propositio per eius subiectum vel predicatum demonstraretur et hoc est impossibile, quia si illa demonstraretur ut est propositio, [quia]⁴⁸ tunc sequitur⁴⁹ hoc impossibile, quod ista propositio esset⁵⁰ antequam esset. Et hoc modo negando casum responderetur ad omnia insolubilia secundi ordinis.

3.3 Exemplum tertii ordinis: sit a ista:

Deus est,

et b ista:

Quelibet istarum est vera,

demonstrando illam:

Deus est

<et c>⁵¹, et sit c ista:

Non quelibet istarum est vera,

demonstrando a et b, sic significando precise. Isto posito, vel b est verum vel falsum. Si verum, igitur ita est sicut totaliter⁵² ista significat et ista totaliter significat quod quelibet istarum est vera, demonstrando a, c, igitur c est verum, igitur ita est totaliter sicut illa significat; et illa si-

45 cum] tamen *Bottin*

46 supponens] suppositionis *Bottin*

47 casus (*corr. cum Bottin*)] *om. ms.*

48 propositio (*corr. cum Bottin*)] propositio quia *ms.*

49 sequitur] ponitur *Bottin*

50 esset (*corr. cum Bottin*)] esse *ms.*

51 et c (*corr.*)] *om. ms. Bottin*

52 sicut totaliter] *inv. Bottin*

gnificat quod non quelibet istarum est vera, demonstrando a et b, igitur altera istarum est falsa; non a, igitur b et sic sequitur quod si b est verum, b est falsum.

Si dicatur quod b est falsum, igitur non quelibet istarum est vera, demonstrando a, b; et c sic precise significat, igitur c est verum. Tunc sic; c est verum et a est verum, igitur quelibet istarum est vera, demonstrando a, c; et b sic precise significat, igitur b est verum; igitur si b est falsum⁵³, b est verum⁵⁴. Et consimiliter probantur omnia insolubilia tertii ordinis.

ad 3.3 Ad illud respondeo et dico negando casum; et datur hec regula: quandoque ponitur casus de insolubili tertii ordinis negatur casus propter istam causam, quia sequitur quod illa propositio esset antequam esset, sicut patet de b et c in casu posito, quia ex quo in b est compositio pro a, c, oportet quod prius naturaliter sint a, c quam b sit, et antequam c propositio sit oportet quod b propositio sit, quia in c fit compositio pro b, et sic sequitur quod tam a tam c quam b essent antequam essent, quod est impossibile.

Et idem universaliter concluditur in omni casu de insolubili tertii ordinis.

3.4 Exemplum quarti ordinis: sit a illa propositio

Deus est,

et b ista:

Homo est animal⁵⁵,

et c illa universalis:

Quodlibet verum est aliquod istorum,

53 falsum (*corr.*)] verum *ms. Bottin*

54 verum (*corr.*)] falsum *ms. Bottin*

55 animal (*corr.*)] asinus *ms. Bottin*

demonstrando a, b sic significando precise. Et ponatur quod non sint plures propositiones quam iste tres. Isto posito, vel c est verum vel falsum.

3.4.1 Si verum, igitur ita est totaliter sicut illa significat et illa significat quod quodlibet verum est aliquod istorum, demonstrando a, b, igitur sic est quod quodlibet verum est aliquod istorum. Tunc sic: quodlibet verum est aliquod istorum, c est verum, igitur c est aliquod istorum. Consequens est contra casum.

3.4.2 Si dicatur quod c est falsum, tunc sic: a verum est et est aliquod istorum, et b verum est <et est>⁵⁶ aliquod istorum, et non est verum quod non est a vel b, igitur quodlibet verum est aliquod istorum, c est verum, igitur c est aliquod istorum. Consequens est⁵⁷ contra casum.

3.4.2.1 Item arguitur sic: quodlibet verum est aliquod istorum et c sic precise significat, igitur c est verum. Consequens contra datum.

Et ita arguitur de omni insolubili quarti ordinis.

ad 3.4 Ad illud respondeo, et admitto casum et concedo quod c⁵⁸ est verum. Et tunc ad argumentum: 'quodlibet verum est aliquod istorum, et c est verum, igitur c est aliquod istorum', ad illud argumentum et consimile duplex potest fieri responsio. Similiter⁵⁹ de quolibet casu insolubili quarti ordinis.

ad 3.4.1 Prima responsio est negare illam consequentiam; et si sic: sequitur quia⁶⁰ est sillogismus in darii, negatur et causa est quia non pro eodem supponit ille terminus 'verum', qui est medius terminus in minori et in

56 et est (*corr. cum Bottin*)] *om. ms.*

57 est] *om. Bottin*

58 c (*corr.*)] *b ms. Bottin*

59 similiter] *similis Bottin*

60 quia (*corr.*)] *quod ms. Bottin*

maiori, quod requiritur. In maiori enim⁶¹ <non>⁶² supponit ille terminus 'verum' pro c, et significat sic quod talis universalis est vera:

Quodlibet verum est aliquod istorum,

demonstrando a et b. Et ideo sicut argumentum non valet:

Quelibet talis, 'Deus est', et quelibet talis, 'Homo est animal', <que>⁶³ significant precise sicut est, est aliquod istorum, sed ista universalis est vera 'Quodlibet verum est aliquod istorum'⁶⁴, demonstrando a et b, igitur <ista>⁶⁵ universalis est aliquod istorum, demonstrando a et b,

nec valet hoc argumentum cum quo convertitur.

3.4.1 Alia est responsio et est illa: distinguendo ex eo quod in singulari significat quod quodlibet verum est aliquod istorum, demonstrando a, b, sic ille terminus 'verum' solum supponit pro a et b, et sic quod non est antecedens; vel ille terminus 'verum' significat a, b, c, et <sic> neganda est minor, quod quodlibet verum est aliquod istorum, demonstrando a, b, quia c verum non est aliquod istorum, demonstrando a, b.

Et consimili modo respondetur ad omnia insolubilia quarti ordinis.

3.5 Exemplum quinti ordinis: pono quod Sortes dicat illam propositionem et nullam aliam:

Sortes dicit falsum,

sic significando precise, que sit a. Isto posito, vel dicit Sortes verum vel falsum.

3.5.1 Si verum et nullam aliam dicit nisi a, igitur a est verum. Tunc sic: a est

61 enim] nam *Bottin*

62 non (*corr.*)] *om. ms. Bottin*

63 que (*corr. cum Bottin*)] *om. ms.*

64 sed ista universalis est vera quodlibet verum est aliquod istorum] *om. Bottin*

65 ista (*corr.*)] *om. ms. Bottin*

verum, igitur totaliter est sicut ipsa significat, et ista significat quod Sortes dicit falsum, igitur Sortes dicit falsum, igitur si Sortes dicit verum, Sortes dicit falsum.

3.5.2 Si conceditur quod Sortes dicit falsum et a sic precise significat, igitur a precise significat <sicut>⁶⁶ est et per consequens a est verum. Et si sic, cum Sortes nichil aliud dicat nisi a, sequitur quod Sortes dicit verum; igitur si Sortes dicit falsum, Sortes dicit verum.

ad 3.5 Ad illud respondeo admittendo casum et concedendo⁶⁷ quod Sortes dicit falsum.

ad 3.5.2 Et tunc ad argumentum quando arguitur:

Sortes dicit falsum et a sic precise significat quod Sortes dicit falsum, igitur a est verum, nego consequentiam. Et ita respondetur ad omne insolubile⁶⁸ quinti ordinis admittendo totum usque quo deveniat ad illud argumentum iam negatum vel consimile ei, quod⁶⁹ (f. 119v) debet negari. Causa quare talis consequentia non valet est ista, quia in a propositione predicatum⁷⁰ est terminus supponens⁷¹ pro complexo et nullus talis terminus potest supponere pro illa cuius est subiectum vel predicatum, ideo oportet quod supponat pro alia⁷² quacumque propositione. <Si pro>⁷³ alia ab illa:

Sortes dicit falsum⁷⁴

supponit illud predicatum 'falsum' in illa propositione:

66 sicut (*corr. cum Bottin*)] *om. ms.*

67 concedendo (*corr. cum Bottin*)] *concedo ms*

68 omne insolubile] *omnia insolubilia Bottin*

69 consimile ei quod (*corr.*)] *consimilem ii ms., consimile illi Bottin*

70 predicatum (*corr.*)] *subiectum ms. Bottin*

71 supponens] *suppositionis Bottin*

72 alia] *om. Bottin*

73 Si pro (*corr.*)] *om. ms. Bottin*

74 falsum] *falsum si Bottin*

Sortes dicit falsum,
falsa est illa propositio:

Sortes dicit falsum,
quia significat Sortem dicere propositionem quam non dicit. Signetur⁷⁵
igitur tunc illa propositio pro qua supponit iste terminus 'falsum', vel
sibi simile, ut illa propositio:

Homo est asinus,
tunc manifestius apparebit defectus argumenti negati, ut si arguitur:

Sortes dicit talem propositionem falsam 'Sortes dicit talem propo-
sitionem "Homo est asinus"', significantem aliter quam est, que
significat precise quod Sortes dicit talem propositionem 'Homo
est asinus', significantem aliter quam est, igitur talis propositio si-
gnificat precise sicut est,
manifestius est quod consequentia non valet quia antecedens est verum
et consequens falsum, posito quod Sortes dicat solum:

Sortes dicit talem propositionem: Homo est asinus,
significando aliter quam significando sic precise, sed primum⁷⁶ argu-
mentum convertitur cum isto, ideo primum argumentum non valet.

3.5.2.1 Aliud est argumentum simile quoddam modo huic argumento, cui dif-
ficilius, ut apparet, respondetur; et hoc est argumentum:

sic <est>⁷⁷ quod Sortes dicit falsum, et a sic precise significat⁷⁸,
igitur <a>⁷⁹ significat precise sicut est.

ad 3.5.2.1 Ad argumentum illud et consimile respondetur concedendo consequen-

75 signetur] significat *Bottin*

76 primum (*corr.*)] ipsum *ms. Bottin*

77 est (*corr.*)] *om. ms. Bottin*

78 precise significat] *inv. Bottin*

79 a (*corr.*)] *om. ms. Bottin*

tiam et dubitando antecedens; et tunc debet queri⁸⁰ utrum idem⁸¹ modus demonstretur⁸² per ly ‘sic’ in maiori et in minori. Et si dicatur quod sic, adhuc queritur utrum in maiori demonstretur⁸³ modus, demonstrando a, significat precise vel non⁸⁴. Si dicatur quod sic, tunc conceditur consequentia et negatur antecedens, scilicet maiorem, scilicet quod ‘sic est quod Sortes dicit falsum’, quia ex quo a est falsum primo modo significans, et nullus primus modus⁸⁵ est, ideo non ⟨est⟩ sic, demonstrando modum⁸⁶ quod a significat.

Sed ⟨si⟩⁸⁷ non demonstratur idem in maiori et minori, tunc non valet consequentia. Si enim⁸⁸ sic arguitur: ‘Sortes dicit falsum, et omni modo quo a significat Sortem dicere falsum, Sortes dicit falsum, igitur a significat precise sicut est’, [et]⁸⁹ ista consequentia est bona, sed minor est falsa, quia Sortes non dicit falsum, quia significat precise Sortem dicere falsum, sicut patet ex predictis.

Consimilem enim⁹⁰ consequentiam oporteret facere in omni insolubili quinti ordinis ad concludendum ipsum esse verum.

3.6 Exemplum sexti ordinis: ponatur quod Sortes credat illam propositionem et nullam aliam:

Sortes decipitur,

significando precise. Isto posito, per idem argumentum probatur illam

80 queri (*corr. cum Bottin*)] *questio ms.*

81 idem (*corr.*)] *iste ms. Bottin*

82 demonstretur] *debetur ms. Bottin*

83 demonstretur] *debetur ms. Bottin*

84 precise vel non (*corr.*)] *pro tali non ms., pluraliter(?) numero Bottin*

85 nullus – modus] *nullo primo modo ms. Bottin*

86 modum (*dub. ms*)] *maiolem Bottin*

87 si (*corr.*)] *om. ms. Bottin*

88 enim] *nam Bottin*

89 est (*corr.*)] *est et ms. Bottin*

90 enim] *nam Bottin*

esse veram et falsam.

ad 3.6 Ad illam respondetur negando casum pro particula ista, quod Sortes credat istam et nullam aliam. Et causa est ista, quia Sortes non potest credere se esse deceptum nisi [aliquis]⁹¹ sciat aliquod esse verum pro nunc quod prius credidit esse falsum, vel quod nunc sciat⁹² aliquid esse falsum quod prius credidit esse verum.

Hoc idem apparet per communem modum⁹³ loquendi, quia si aliquis dixerit se esse deceptum et queritur ab eo:

Quare es tu deceptus?,

respondetur sic:

Ego credidi sic vel sic,

referendo⁹⁴ actum suum ad prius creditum. Et nullus dicit se decipi propter actum credendi quem habet, sed propter actum credendi⁹⁵ quem habuit. Et isto modo fiet responsio ad omnia insolubilia sexti ordinis.

Expliciunt insolubilia magistri Johannis de Dulminton sub compendio accepta secundum usum Oxonie.

91 nisi (*corr. cum Bottin*)] nisi aliquis *ms.*

92 sciat (*corr.*)] stat *ms. Bottin*

93 idem apparet per communem modum (*corr.*)] apparet per idem modum *ms. Bottin*

94 referendo (*corr.*)] referendi *ms. Bottin*

95 credendi (*corr.*)] demonstrandi *ms. Bottin*

Anonymous, An Epitome of John Dumbleton's Solution to Insolubles according to the Oxford usage

- 1.0 I intend in the present ⟨work⟩ to divide insolubles or scenarios (*casus*) of insolubles into six groups (*ordines*). Its group more readily provides for the major part the response to any insoluble scenario proposed.
- 1.1 The first group is when it is assumed that there is only one proposition of some sort, whose subject or predicate supposits for a (propositional) complex (*complexum*).⁹⁶
- 1.2 The second group assumes that some proposition is referred to by its own subject or predicate.
- 1.3 The third group is when it is assumed that some proposition exists whose subject or predicate supposits for some proposition or propositions which will be required to exist naturally-before its subject or predicate supposits for those propositions; nor indeed could that proposition or those propositions for which the subject or predicate of the proposition supposits exist naturally-before that proposition, whose subject or predicate supposits for this proposition or those propositions, existed, so that briefly in every scenario of the third group it follows that the proposition exists before it exists.
- 1.3.1 E.g., let B be the proposition
 God exists,
and let A be
 Each of these is true,

⁹⁶ See, e.g., NUCHELMANS 1973, 300: “*complexum*: combination of words, esp. statement-making utterance”; also §§ 11.1.2–3 and ch. 12 *passim*.

referring to propositions B and C, where C is the proposition

Not all of these are true,

referring to propositions A and B. Then the subject and predicate in A supposit for propositions B and C and so for A to exist it is required that propositions B and C, for which A supposits, existed before. But C can only exist if propositions A and B existed before, since ⟨its⟩ subject and predicate supposit for A and B, and so it follows that A would exist before it existed. And the same follows for every insoluble of the third group.

1.4 The fourth group wholly agrees with the third group except that it is not required that an insoluble of the fourth group exists before a proposition exists for which the subject or predicate of that insoluble supposit.

1.5 The fifth group is when it is assumed that someone says, utters, hears or sees only one proposition whose subject or predicate supposits for a (propositional) complex, as in assuming that Socrates says this proposition and no other proposition:

Socrates says a falsehood,

or sees this and no other:

Socrates sees a falsehood.

1.6 The sixth group is when it is assumed that someone believes only one proposition whose subject or predicate supposits for a (propositional) complex, as in assuming that Socrates believes this proposition and no other:

Socrates is deceived.

2.0 Now among these six groups no scenario should be admitted unless it

is a scenario of the fourth or fifth group.

- 2.1 The first group: 'A truth exists'; 'A falsehood exists'; 'No truth exists'; 'No falsehood exists'; 'A proposition exists'; 'A necessity exists'; 'A possibility exists'; 'An impossibility exists'; 'Every universal proposition is false'; 'No universal proposition is false'; 'Only an exclusive is false'; 'No exceptive is false but this', referring to itself.⁹⁷
- 2.2 The second group: 'It is not as this proposition signifies'; 'This proposition is false and it signifies other than it is'; 'This proposition is not true "Through no time it was"'; 'The true contradictory of this is true'; 'God exists and only the first conjunct of this conjunction is true'; 'God exists and every conjunction is false'; 'A man is an ass and no conjunction is true'.
- 2.3 The third group: 'Each of these is true', referring to A and C; and 'Not all of these are true', referring to A and B;⁹⁸ and 'Each A is similar to B, 'Socrates is deceived or the contradictory of this disjunction is true'; 'God does not exist or this disjunction is true'.
- 2.4 The fourth group: 'Each truth is one of these'; 'These are all the truths'; 'God exists and a man is an ass'; 'Only one of these is true'; 'God exists and only this proposition is true'; 'A man exists and only this is a proposition'.
- 2.5 The fifth group: 'Socrates says a falsehood'; 'Socrates does not say a truth'; 'Socrates is a liar'; 'Socrates breaks his oath'; 'Socrates speaks a falsehood'.
- 2.6 The sixth group: 'Socrates is deceived'; 'Socrates is not deceived';

⁹⁷ This example seems to fit better under the second group. Perhaps it was included in the first group as a result of a scribal error.

⁹⁸ The letters A, B and C appear to correspond to the example in § 3.2.3 below.

'Someone is deceived'; 'Someone is deceived and Socrates is he'.

3.0 To all the insolubles of the first group there is one and the same response, and for all of them and similar ones the proof is the same.

3.1 E.g., assume that A is the proposition:

A falsehood exists,

signifying only like that, and that there is no other proposition than it. Assuming this, A is either true or false; if it is said that it is true, then it is wholly as it signifies and it only signifies that a falsehood exists, so a falsehood exists. And since there is no other proposition than A, it follows that A is false, so if A is true, it follows that A is false. If it is said that A is false, then ⟨one argues⟩ in this way: A is false, therefore it is not as it signifies and it signifies only like that, therefore ⟨A is true, and so⟩ if A is false it follows that A is true. And one can argue in a similar way for every insoluble of the first group.

3.2 For the solution one should reply by rejecting the scenario whether it is an insoluble of the first group or the second group or the third, but a different reason should be assigned for the impossibility of the scenario in the one than in the other.

3.2.1 The reason why a scenario of an insoluble of the first group is impossible is that the ⟨insoluble⟩ proposition can only exist if there is another proposition, since this proposition must conform to the subject or predicate of the insoluble, since the subject or predicate of the ⟨insoluble⟩ proposition is a term suppositing for a (propositional) complex, as was argued earlier.⁹⁹

3.2.2 The reason why the scenario of an insoluble of the second group is im-

⁹⁹ That is how the first group is defined.

possible is that it is proposed that this very proposition is referred to by its subject or predicate, and this is impossible. For if it is referred to insofar as it is a proposition, then the impossibility follows that this proposition existed before it existed. And one should respond in this way by rejecting the scenario in all insolubles of the second group.

3.3 An example of the third group: let A be:

God exists,

and B:

Each of these is true,

referring to ⟨both⟩:

God exists

and C, and let C be:

Not all of these are true,

referring to A and B, signifying only like that. Assuming this, either B is true or false. If true, then it is wholly as it signifies and it wholly signifies that each of these is true, referring to A and C, therefore C is true, therefore it is wholly as ⟨C⟩ signifies, and it signifies that not all of them are true, referring to A and B, therefore one of them is false; not A, therefore B, and thus it follows that if B is true, B is false.

If it is said that B is false, then not all of these are true, referring to A and B, and C only signifies like that, so C is true. Then ⟨one argues⟩ in this way: C is true and A is true, therefore each of these is true, referring to A and C; and B only signifies like that, so B is true, therefore if B is false, B is true and all insolubles of the third group are proved similarly.

ad 3.3 To this I respond by rejecting the scenario and I give this rule: whenever a scenario of an insoluble of the third group is proposed, the scenario should be rejected for this reason, that it follows that the proposition

would exist before it existed, as is clear regarding B and C in the scenario posited. Because from the fact that in B there is a composition about A and C, it is required that A and C exist naturally-before B exists, and before proposition C exists, it is necessary that proposition B exists, because in C there is composition about B, and thus it follows that A and C and B existed before they existed, which is impossible.

And the same is universally concluded in every scenario of an insoluble in the third group.

3.4 An example of the fourth group: let A be the proposition:

God exists,

and B the proposition:

A man is an animal,

and C the universal proposition:

Every truth is one of these,

referring to A and B, signifying only like that. And assume that there are no more propositions than these three. Assuming this, either C is a truth or a falsehood.

3.4.1 If $\langle C \text{ is} \rangle$ a truth, then it is wholly as it signifies, and it signifies that every truth is one of these, referring to A and B, hence it is such that every truth is one of these. Then $\langle \text{one argues} \rangle$ in this way: every truth is one of these, C is a truth, therefore C is one of these. The conclusion is contrary to the scenario.

3.4.2 If it is said that C is a falsehood, then $\langle \text{one argues} \rangle$ in this way: A is a truth and it is one of these, and B is a truth and it is one of these, and there is no truth that is not A or B, so every truth is one of these, C is a truth, therefore C is one of these. The conclusion is contrary to the scen-

ario.

3.4.2.1 Again, one argues in this way: every truth is one of these and C signifies only like that, therefore C is true. The conclusion is contrary to what was given.

And one argues in this way for every insoluble of the fourth group.

ad 3.4 To that I respond by admitting the scenario and by granting that C is a truth. And then to the argument: 'every truth is one of these, and C is a truth, therefore C is one of these', to that argument and similar ones, a response can be made in two ways, <and> likewise for any insoluble scenario of the fourth group.

ad 3.4.1 The first response is to deny the inference;¹⁰⁰ and if <one argues> in this way I deny that it is a syllogism in Darii and the reason is that the term 'truth', which is the middle term in the minor and in the major premise, does not supposit for the same thing, which is required <for validity>. For in the major premise the term 'truth' does not supposit for C and signifies in such a way that this universal is true

Every truth is one of these,

referring to A and B. Hence, just as <this> argument:

100 This seems to be the response of Walter Segrave: see SEGRAVE IN PREPARATION, § ad 6.1.1: "Then to the first <paralogism> I reply that assuming this scenario, this is true: 'Every truth is one of these', and I deny the <validity of> the inference <in Darii> 'Every truth is one of these, this is a truth, therefore this is one of them', for the middle term varies because in the minor <premise> it supposits for this truth: 'Every truth is one of these', but in the major <it does> not. And so the conclusion does not follow <from the premises>. For the meaning of the major is: 'Every truth other than <the major premise> (or what is convertible with it and so on for others for which <the subject> does not supposit), is one of these, and <the subject> does not supposit for <the major premise>'." (Ad primum igitur dico, illo casu posito, quod hec est vera: Quodlibet verum est aliquod istorum, et nego consequentiam: Quodlibet verum est aliquod istorum, hoc est verum, ergo hoc est aliquod istorum. Medium enim variatur; in minori namque supponit pro hoc vero: Quodlibet verum est aliquod istorum, sed in maiori non. Et ita non sequitur conclusio. Unde sensus maioris est: Quodlibet verum aliud ab hoc, vel convertibile cum eo et ita de aliis pro quibus non supponit, est aliquod istorum, et non supponit pro hoc.)

Each occurrence of ‘God exists’ and each occurrence of ‘A man is an animal’ – which signify only as it is – is one of these, but the universal ‘Every truth is one of these’ is true – referring to A and B – therefore the universal is one of these, referring to A and B, is not valid, neither is the argument ⟨in 3.4.1⟩ valid with which it converts.¹⁰¹

aliter ad 3.4.1 The other response is this: by distinguishing ⟨two cases⟩ for the singular premise (i.e. ‘C is a truth’): (i) where C signifies truly that every truth is A or B (so ‘truth’ supposits only for A and B), and then the minor is not sufficient for (i.e. antecedent to) the conclusion; (ii) where C signifies falsely that every truth is A, B or C (so ‘truth’ supposits for A, B and C), and then the minor is false and so is denied.

And one should respond in a similar way to every insoluble of the fourth group.

3.5 An example of the fifth group: assume that Socrates says this proposition and no other

Socrates says a falsehood,
signifying only like that, call it A. Assuming this, either Socrates says a truth or a falsehood.

3.5.1 If ⟨Socrates says⟩ a truth and he says nothing else but A, then A is a truth. Then ⟨one argues⟩ in this way: A is a truth, therefore it is wholly as it signifies and it signifies that Socrates says a falsehood, therefore Socrates says a falsehood, therefore if Socrates says a truth, Socrates

¹⁰¹ The first response is to say that the argument is invalid, having true premises and false conclusion, since there is a fallacy of accident, that is, of variation of supposition in the major and minor premises; the second response is to accept that the argument is valid, but that one premise is false; either the major (else there is a fallacy of four terms), or the minor.

says a falsehood.

3.5.2 If it is granted that Socrates says a falsehood and A signifies precisely like that, therefore A signifies precisely as it is and consequently A is a truth. And if so, since Socrates says nothing except A, it follows that Socrates says a truth, therefore if Socrates says a falsehood, Socrates says a truth.

ad 3.5 I respond to that ⟨example⟩ by admitting the scenario and I grant that Socrates says a falsehood.

ad 3.5.2 And then to the argument, when one argues: ‘Socrates says a falsehood and A signifies precisely that Socrates says a falsehood, therefore A is a truth’, I deny the inference. And one may respond in this way to every insoluble of the fifth group by admitting the whole ⟨argument⟩ until we reach the argument just denied or one like it, which should be denied. The reason why this inference is not valid is this, that in proposition A the predicate is a term suppositing for a (propositional) complex and no such term can supposit for a proposition of which it is the subject or predicate, hence it is required that it supposits for some other proposition. If the predicate ‘falsehood’ in that proposition ‘Socrates says a falsehood’ supposits for a proposition other than

Socrates says a falsehood,

the proposition

Socrates says a falsehood

is false, because it signifies that Socrates says a proposition which he does not say. Then let the proposition for which the term ‘falsehood’ supposits or something similar to it be specified, e.g. the proposition

A man is an ass,

then the defect in the argument denied will appear more manifest. E.g.,

if one argues

Socrates says this false proposition:

Socrates says this proposition: 'A man is an ass'

(signifying other than it is), which signifies precisely that Socrates says the proposition: 'A man is an ass' (signifying other than it is), therefore, that proposition signifies precisely as it is,

it is more manifest that the inference is not valid because the premises are true and the conclusion false, assuming that Socrates says only:

Socrates says the proposition: 'A man is an ass'

(signifying other than by signifying precisely in that way). But the first argument is equivalent to this one, therefore the original argument (in § 3.5.2) is not valid.

3.5.2.1 There is another argument similar in some ways to this argument, to which one responds with more difficulty, it seems, and this is the argument:

It is in such a way (*sic*) that Socrates says a falsehood and A signifies precisely in such a way (*sic*), therefore A signifies precisely in such a way (*sicut*) as it is ⟨in reality⟩.

ad 3.5.2.1 I respond to that argument and similar ones by granting the inference and doubting the premise; and then it should be asked ⟨1⟩ whether the same way ⟨of signifying⟩ is referred to by ⟨the adverbial phrase⟩ 'in such a way (*sic*)' in the major premise and in the minor. And if one says that it is, then I ask ⟨2⟩ whether in the major premise the way of ⟨of signifying⟩ referred to ⟨by 'in such a way'⟩, referring to A, is signifying precisely or not. If it is said ⟨in response⟩ to ⟨2⟩ that it is, then the inference is granted and the premise is denied, i.e., the major premise, namely, that 'it is in such a way that Socrates says a falsehood', because,

from the fact that A is false signifying in the first way and there is no first way, for that reason it is not in such a way (*sic*), referring to the way in which A signifies.¹⁰²

But if ⟨it is said to ⟨1⟩ that⟩ the same ⟨way of signifying⟩ is not referred to in the major premise and the minor, then the inference is not valid. For, if one argues in this way:

Socrates says a falsehood and in every way in which A signifies that Socrates says a falsehood, Socrates does say a falsehood, therefore A signifies only (*precise*) as it is ⟨in reality⟩,

this inference is good, but the minor is false because Socrates does not say a falsehood insofar as A signifies only (*precise*) that Socrates says a falsehood, as is clear from what has been said.

In fact, a similar inference should be drawn in ⟨the case of⟩ every insoluble of the fifth group to deduce that it is true.

3.6 An example of the sixth group: assume that Socrates believes this proposition and no other:

Socrates is deceived,

signifying only in that way. Assuming this, the same argument proves that it is true and false.

ad 3.6 To that I respond by rejecting the scenario for this part, that Socrates believes this and no other. And the reason is this, that Socrates cannot believe himself to be deceived unless he knows something to be true now that he earlier believed to be false or that now he knows something to be false that earlier he believed to be true.

¹⁰² The Latin is puzzling syntactically and so its meaning is somewhat unclear. Perhaps the author does not consider the other leg of the dilemma ('or not') since that option falls under the second option in the first question. We are grateful to an anonymous reviewer for this clarification.

The same thing appears through the usual way of speaking because if someone said that he was deceived, and he is asked:

Why are you deceived?,

he responds in this way:

I believed so and so,

referring his act to an earlier belief. No one says he is deceived through an act of believing that he has, but through an act of believing that he had. And this should be the response made to every insoluble of the sixth group.¹⁰³

Here end the insolubles of master John of Dumbleton in an epitome made according to the Oxford usage.

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103 § 3.6 repeats almost verbatim a similar passage in ch. 21 of Dumbleton's *Summa Logicae*.

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EPISTEMIC SOPHISMS, *CALCULATORES* AND JOHN MAIR'S CIRCLE

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Abstract: This paper focuses on the early sixteenth-century epistemic logic developed by John Mair's circle and discusses iterated epistemic modalities, epistemic closure and Bradwardinian semantics related to the logic of epistemic statements. These topics are addressed as part of setting up and solving epistemic sophisms based on traditional scenarios which can be traced back to fourteenth-century British epistemic logic. While the ultimate source for the debate appears to be the second chapter of William Heytesbury's *Regule solvendi sophismata*, the immediate source is the Italian editorial, commentarial and philosophical tradition, notably Paul of Venice and Cajetan of Thiene.

Keywords: Oxford Calculators; John Mair's circle; epistemic logic; epistemic modalities; sophisms; scholastic logic.

1. Introduction

Around 1330 the solution to epistemic sophisms grew into a sovereign genre, commonly referred to as '*De scire*' or '*De scire et dubitare*', through the works of authors such as William Heytesbury, whose treatise would influence epistemic logic all the way to the mid-sixteenth century. Heytesbury's logic of believing, knowing and doubting is predominately based on the Aristotelian distinction between composed and divided sense, which captures different readings of doxastic and epistemic statements and relevant inference rules.

* This publication is an outcome of the *Scholastic physics in the era of the scientific revolution* project, registration number 20-05855S, supported by The Czech Science Foundation (GAČR) and delivered by the Institute of Philosophy of the Czech Academy of Sciences. I have benefited from the comments of Stefano Caroti and the referees. By primarily focusing on a different corpus of sixteenth-century texts, this paper elaborates on a series of papers, "Scholastická logika 'vědění'" ["The Scholastic Logic of 'Knowledge'"], currently being published in *Studia Neoaristotelica*.

His treatise *De scire et dubitare* from *Regule solvendi sophismata* consists of a series of epistemic sophisms, the doctrinal core discussing logical independence of *de re* and *de dicto* contexts and iterated modalities, and the solution to the opening sophisms. All sophisms are developed as games of *obligationes*, i.e., as disputations based on a posited scenario, where the key issue is the compatibility of knowledge and doubt.¹ These scenarios would later become widely used and, quite possibly, Heytesbury himself inherited some of them from his forebears (such as Richard Kilvington).² Together with explicit references, included in the main text or added in the margins, the common practice of reusing the scenarios of epistemic sophisms and the common form of such sophisms makes both the continuity with and all deviations from the original sources relatively easy to detect.

This paper focuses on the development of the fourteenth-century tradition of British epistemic logic in the sixteenth-century John Mair's circle in Paris. The primary corpus includes texts by Jerome Pardo (d. 1502), a teacher and collaborator of John Mair, John Mair (1467–1550), Mair's student Antonio Coronel (d. around 1521), Gaspar Lax (1487–1560), who was a student of John Mair and a teacher of Juan de Celaya, and Juan de Celaya (ca. 1490–1558), a student of Jan Dullaert, Gaspar Lax and possibly John Mair, and a teacher of Domingo de Soto.³ Three groups of problems will be addressed: iterated epis-

1 See WILLIAM HEYTESBURY 1494(1) (the critical edition is not available, for an English translation see WILLIAM HEYTESBURY 1988), WILLIAM HEYTESBURY 1494(2) analysed in STUMP 1989, BOH 1993, HANKE 20018(1), HANKE forth.(1), and HANKE 2021 (containing a critical edition of Pseudo-Heytesbury's *Casus obligationis*). For a basic overview of *obligationes*, see SPADE, YRJÖNSUURI 2020, for *obligationes* in the circle of the Oxford Calculators, see YRJÖNSUURI 1990. Finally, there is ARRIBAS 1993, which is relevant to the genre of *obligationes* in John Mair's circle. The term 'scenario' as a translation of 'casus' or 'positum', which means the same in this context, is from READ 2020(1).

2 See KILVINGTON 1990, 119–151, analysed in (among others) STUMP 1989, 222–231 and BOH 1993, 62–77.

3 For biographical and bibliographical data see the following footnote. The quotations rely on working transcriptions; the orthography and punctuation has been adapted,

temic modalities pertaining to the debate on whether one can doubt one's own knowledge; epistemic closure pertaining to epistemological debates; and Bradwardinian sentential semantics. The body of epistemic sophisms introduced by the aforementioned authors is by no means limited to those discussed below, but these in particular are interesting even from the modern perspective, as a consequence of being directly relevant to the formal properties of epistemic modalities, and can be motivated in an intuitive way. The study is rooted in a growing body of research into three related issues, namely post-medieval logic in general,⁴ scholastic doxastic and epistemic logic,⁵ and epistemology in John Mair's circle⁶. It is intended to contribute to each of these fields to some extent.

2. Iterated Epistemic Modalities

Iterated epistemic modalities were a standard issue of fourteenth-century epistemic logic and were comprehensively researched by Ivan Boh.⁷ Two instances of debating these topics by Jerome Pardo and Gaspar Lax will now be

rare and minor corrections are limited to obvious errors and are not indicated.

4 The seminal work in the field is ASHWORTH 1974(2); for more recent overviews by the same author, ASHWORTH 2008 and ASHWORTH 2016. For an introductory publication which covers the authors pertaining to John Mair's circle, see BROADIE 1987, BOH 2001 or LAGERLUND 2017. As for the more specific material, Alexander Broadie authored multiple publications on John Mair's circle, including BROADIE 1983 and BROADIE 1985; for bibliography, see DURKAN 1950, LOHR 1975 and LOHR 1978.

5 The most general sources are BOH 1993 and BOH 1997. For epistemic logic analysed as related to *sophismata* and *obligationes*, see STUMP 1989, 215–249, YRJÖNSUURI, COPOCK 2016, 277–280, and HANKE 20018(1), 2018(2), forth.(1) and forth.(2).

6 See BROADIE 1993 and BROADIE 1995, KARGER 2009, LAGERLUND 2019, LONGEWAY 2009.

7 See BOH 1993, 73–76 and 111–112, and BOH 1984 which focus on William Heytesbury, Peter of Mantua, and Cajetan of Thiene. Other sources pertaining to this tradition, notably John Wyclif, John of Holland, Paul of Venice, Paul of Pergula, and Mengho Bianchelli were analysed in HANKE 20018(1) and HANKE 2018(2). There is still a large number of unanalysed relevant texts, such as JOHN HUNTER 1999, 418–445, presumably one of Paul of Venice's sources, and a number of unedited treatises related to Heytesbury's treatise (which could turn out interesting despite their relatively minor influence).

presented. Despite their respective specific contexts, they display notable fourteenth-century influences.

2.1 Jerome Pardo: *Medulla dyalectices* (1500/1505)

Pardo addresses iterated epistemic modalities in the seventh chapter of his *Medulla dyalectices*. The chapter addresses the truth conditions of statements as related to the appellation of terms, including ‘*appellatio rationis*’ in epistemic statements; note the underlying use of the terminist semantics.⁸ As part of a discussion of relevant logical rules, Pardo introduces the axiom **K** of epistemic logic: knowledge distributes over implication or, in the original phrasing, if an inference is valid and an agent knows that it is valid and knows that the antecedent holds, then the agent also knows that the consequent holds. Pardo notes here that the rule might include an additional requirement that the agent is not diverted from contemplating the problem, but ultimately does not regard such extension as quite necessary. Despite the terminist phrasing, the principle is viewed as pertaining to the genre of consequences.⁹ Afterwards, Pardo introduces a *dubium* whether the same statement can simultaneously be a matter of knowledge and doubt or conjecture and gives a negative answer, followed by counterexamples and their analysis.¹⁰

8 For the widely discussed notion of *appellatio rationis*, see (e.g.) NUCHELMANS 1988, BOH 1993, 85–86, and the recent PANACCIO 2012; for *appellatio rationis* as related to logical omniscience, see HANKE forth.(1). For a general overview of terminism, see READ 2019.

9 “Supponamus ulterius pro materia deducenda aliquas regulas generales consequentiarum. Prima regula: si aliqua consequentia est bona et scita esse bona et antecedens est scitum ab aliquo (capiendo antecedens pro significato), consequens est scitum ab eodem (capiendo consequens pro significato). (...) Tamen posset dici, licet non sit necesse, quod regula predicta sic intelligitur, quod scito antecedente et scita bonitate consequentie, si voluntas non divertat intellectum a consideratione conclusionis, scitur etiam conclusio, et loquor de antecedente totali,” JEROME PARDO 1505, fol. 97ra–rb.

10 “Quibus suppositis sine ampliori declaratione quero tale dubium: utrum eadem propositio sit dubia et scita vel scita et opinata, vel magis proprie loquendo an idem significatum secundum eandem propositionem sit scitum et dubium vel scitum et opinatum.

The eighth counterexample argues that since it is possible that someone knows something while merely conjecturing that he knows it, the same thing can be known and conjectured, and, as a consequence, known and doubted.¹¹ The proof breaks down into the proof of the assumption and the proof of the inference. According to the first sub-proof, the scenario that someone knows that Socrates is running while conjecturing that he knows it is consistent. The justification dismantles one particular threat to that scenario's consistency, namely the inference to the conclusion that the agent knows that he knows that Socrates is running via the axiom of positive introspection ('if **p** knows that *X* is the case, then **p** knows that **p** knows that *X* is the case'). This principle is dismissed by pointing out the independency of direct and reflexive cognitive acts.¹² The second sub-proof argues that the scenario entails the coexistence of knowledge and conjecture with respect to the same problem. The knowledge part is presupposed in the scenario. The conjecture part follows from the assumption that the agent conjectures that he knows that Socrates is running: whoever conjectures that he knows that Socrates is running *ipso facto* conjectures that such knowledge is factually correct. That completes the ar-

Respondeo talem conclusionem ponendo: non est possibile idem significatum totale propositionis esse scitum et opinatum ab eodem, scientia et opinione precise representantibus tale significatum, et hoc sive teneatur primus modus dicendi de complexe significabilibus sive non," JEROME PARDO 1505, fol. 97rb. Note the mentioning of propositional ontology as well as its irrelevance to the problem.

11 "Octava instantia: possibile est quod opinaris te scire Socratem currere et tamen quod scias Socratem currere, quo facto sequitur quod possibile est te scire Socratem currere et opinari Socratem currere, et per consequens idem est scitum et dubium," JEROME PARDO 1505, fol. 101rb. Note that first edition, i.e., JEROME PARDO 1500, of *Medulla dyalectices* has a different numbering but the text is, in this particular regard, identical. The same is true for Pardo's text discussed below in section 4.

12 "Primum patet, videlicet quod possibile sit te scire Socratem currere et simul opinari te scire Socratem currere, quia non sequitur: 'ille scit Socratem currere, ergo scit se scire Socratem currere'. Non enim necesse est quod habito illo actu, habeatur actus reflexus. Ideo non apparet repugnantia quod scientia qua scis Socratem currere stet cum opinione qua opinaris te scire Socratem currere," JEROME PARDO 1505, fol. 101rb.

gument.¹³ The reason seems to be that knowledge breaks down into the existence and factual correctness of a mental act.¹⁴ To conjecture that I know X breaks down into conjecturing that I assent to X and to conjecturing that X is the case, presumably since conjecture is closed under conjunction elimination.

The proof of the corollary (that the same thing can be known and doubted) is not introduced explicitly. It is not a direct implication of the scenario, since conjecturing excludes rather than entails doubting.¹⁵ However, one could replace ‘to conjecture’ with ‘to doubt’ in Pardo’s argument: the scenario that someone knows that X is the case and doubts whether he knows that X is the case is consistent, whence the same thing can be simultaneously known and doubted. The proof decomposes into two sub-proofs. The first sub-proof (that the scenario is consistent) secures the consistency of the scenario by rejecting the axiom of positive introspection (which is, by assumption, the most severe problem). The second sub-proof (that the inference is valid) argues that the scenario entails that someone simultaneously knows and doubts that Socrates is running, since whoever doubts that he knows that X is the case *ipso facto* doubts that X is the case. For that inference to be legiti-

13 “Sed probatur secundum, videlicet quod ad aliquem scire Socratem currere et opinari se scire Socratem currere sequatur quod scit Socratem currere et quod opinatur Socratem currere. Nam primo habetur quod scit Socratem currere. Sed quod opinetur Socratem currere, probo, quia bene sequitur: ‘*opinatur se scire Socratem currere, ergo opinatur ita esse sicut per illam scientiam significatur*’. Et sic habetur quod scit Socratem currere et opinatur Socratem currere, quod erat probandum,” JEROME PARDO 1505, fol. 101rb–va. Note that these arguments have numerous presuppositions regarding the logic of epistemic and doxastic verbs, which makes them vulnerable to criticism.

14 “Ideo posset dici quod propositio de ‘*scio*’ exponitur per unam copulativam in qua una partium ostendet existentiam illius qualitatis, altera vero exprimet quod ita est sicut per talem qualitatem significatur, ut ista propositio: ‘*scio Socratem currere*’ posset sic exponi: ‘*hec scientia est (demonstrando illum assensum) et Socrates currit*’,” JEROME PARDO 1505, fol. 101va.

15 The reason is that conjecturing entails assenting to a sentence, whereas doubting a sentence entails neither assenting to nor dissenting from it (see JEROME PARDO 1505, fol. 96ra–va). As a result, the two acts are mutually exclusive.

mate, one has to assume that whoever doubts whether he knows that X is the case *ipso facto* doubts whether X is the case, because knowledge breaks down into etc.

Pardo summarises his solution to this counterexample in a series of theses, the first two of which will now be presented: first, it is not absolutely impossible or inconsistent that someone conjectures or doubts that he knows something; second, it is consistent to assume that someone knows something while conjecturing or doubting that he knows that.

The first thesis regards as consistent the scenario that someone conjectures that he knows something. The proof is that if the scenario did imply a contradiction, it would most likely be this one: '*p* knows that Socrates is running and *p* fails to know that Socrates is running'. However, the inference: '*p* conjectures that *p* knows that Socrates is running; therefore, *p* knows that Socrates is running' is invalid, since conjectures are not veridical. Similarly, the scenario that someone doubts that he knows something is considered consistent.¹⁶ This argument is problematic for at least two reasons. First, there are reputable medieval proofs that such a scenario is inconsistent, which makes the argument weak in terms of historical awareness.¹⁷ Second, it is meaningless to claim that a scenario is consistent unless the relevant rules of inference are

16 "Prima propositio: non est absolute impossibile quod aliquis opinetur se scire Socratem currere. Probat, quia ad hoc non sequitur aliqua contradictio, quia maxime sequeretur ista quod sciret Socratem currere et quod non sciret Socratem currere. Sed illa non sequitur, nam ego dicam quod non scit Socratem currere, unde non valet hec consequentia: '*tu opinaris te scire Socratem currere, ergo tu scis Socratem currere*'. Nam possibile est quod opineris aliter quam est, ut videlicet opineris me scire Socratem currere, et tamen non sciam Socratem currere, quemadmodum faciunt presumptuosi, qui dicunt opinantes secundum unam significationem. Et per idem patet quod non est repugnantia quod aliquis dubitet se scire Socratem currere," JEROME PARDO 1505, fol. 101va.

17 For such arguments proposed by Heytesbury, John of Holland and Paul of Venice (which appear to be representative of a broader corpus), see BOH 1984, BOH 1993, 67–76 and 111–112 and HANKE 2018(1), 150–158, 164–169 and HANKE 2018(2), 214–233. Incidentally, some of the historical arguments derive a different form of contradiction.

specified and in this particular case, a self-contradiction can easily be derived from the scenario in a sufficiently strong logical system.¹⁸

The second thesis, which is presented as probable, is that the scenario that someone knows that Socrates is running while conjecturing that he knows that Socrates is running, is inconsistent, since the person would at the same time know that Socrates is running and conjecture that Socrates is running. Pardo argues that whoever conjectures that he knows that *X* is the case, conjecturally assents to the statement: '*X is the case and the knowledge (that X is the case) exists*' implicitly (*virtualiter*), as this conjunction is the *explicans* of epistemic statements.¹⁹ Also, whoever assents to a conjunction conjecturally, *ipso facto* assents to its sub-formulas conjecturally. The function of virtual con-

18 The scholastic arguments aside, let us outline such argument in the axiomatic system S5 (as defined in, e.g., HUGHES, CRESSWELL 1996, 51-70), validating both positive and negative introspection, i.e., both 'if *p* knows that *X* is the case, then *p* knows that *p* knows that *X* is the case' and 'if *p* fails to know that *X* is the case, then *p* knows that *p* fails to know that *X* is the case'. Furthermore, let us assume that if someone doubts that *X* is the case, then he does not know whether *X* is the case, i.e., he fails to know that *X* is the case and he fails to know that *X* is not the case. Now let us assume as a hypothesis that *p* doubts whether he knows that *X* is the case. As a result, *p* fails to know that he knows that *X* is the case and *p* fails to know that he fails to know that *X* is the case. In that case, either *p* knows that *X* is the case, or he fails to know that *X* is the case (a tautology). That splits the scenario into two sub-hypotheses. First, the hypothesis that *p* knows that *X* is the case: if *p* knows that *X* is the case, then *p* knows that he knows that *X* is the case (by positive introspection). However, the scenario entails that *p* fails to know that he knows that *X* is the case (see above). A contradiction. Second, the hypothesis that *p* fails to know that *X* is the case: if *p* fails to know that *X* is the case, then *p* knows that he fails to know that *X* is the case (by negative introspection). However, the scenario entails that *p* fails to know that he fails to know that *X* is the case (see above). A contradiction. As both mutually exclusive hypotheses exhaustively develop the original scenario, that is self-contradictory (since it is self-contradictory on both sub-hypotheses).

19 "Pro solutione advertendum est quod ille terminus '*scientia*' supponit pro quadam qualitate in anima existente connotando quod ita sit sicut per ipsam significatur (...) Ideo posset dici quod propositio de scio exponitur per unam copulativam in qua una partium ostendet existentiam illius qualitatis, altera vero exprimet quod ita est sicut per talem qualitatem significatur, ut ista propositio: '*scio Socratem currere*' posset sic exponi: '*hec scientia est (demonstrando illum assensum) et Socrates currit*'. Ex quo patet quod assentire se scire Socratem currere est assentire illi copulative: '*hec scientia est et Socrates currit*,'" JEROME PARDO 1505, fol. 101va.

jecture in this argument suggests that conjectural assent is closed under entailment.²⁰

As an alternative, Pardo contemplates the position ascribed to an anonymous *doctor*, who claims that one's will can force a dissent to the law of non-contradiction or an assent to a self-contradiction. Similarly, someone can assent to a conjunction without assenting to its sub-formulas.²¹ Pardo ultimately considers his own view more probable and offers two restatements thereof. First, he replaces 'conjecturing' with 'doubting': it is impossible to know that Socrates is running while doubting that knowledge. Second, it is impossible to have evidence for a statement while doubting that one is in possession of such evidence.²² Pardo does not elaborate, but he could be open to restating the proof of the second thesis as follows: doubting that one knows that X is the case entails doubting that X is the case, which makes the scenario inconsistent. The most controversial step is, again, the assumption that doubt is closed under entailment (or, at the very least, under conjunction elimination), together with assuming that knowledge is veridical (the axiom

20 "Secunda propositio probabilis: non stat quod aliquis sciat Socratem currere et opinetur se scire Socratem currere. Probat, nam ex illo sequitur quod ille scit Socratem currere et opinatur Socratem currere. Nam quod ille scit Socratem currere patet ex supposito. Sed quod ille opinetur Socratem currere patet, quia opinari se scire Socratem currere est opinari virtualiter istam propositionem: '*Socrates currit et hec scientia est*'. Qui autem opinatur aliquam copulativam, opinatur quamlibet et eius partem, ergo ille opinatur quod Socrates currit," JEROME PARDO 1505, fol. 101va.

21 "Oppositum tamen istius propositionis potest defendi proterve maxime si teneatur opinio unius doctoris quod voluntas prave affectata cum aliquibus motivis potest dissentire primo principio, ut puta quod voluntas posset facere quod intellectus unico assensu assentiret isti copulative: '*Socrates currit et Socrates non currit*', nulli tamen parti. Oporteret enim assentire assensu proprio illi parti. Et non minus videtur inconveniens quod quis opinetur istam copulativam: '*hec scientia est et Socrates currit*' et non partes seorsum," JEROME PARDO 1505, fol. 101va.

22 "Tamen probabilius est tenere illam secundam propositionem quam eius oppositum, per quam patet etiam quod hec copulativa est impossibilis: '*aliquis scit Socratem currere et dubitat se scire Socratem currere*'. Et consimiliter impossibile est quod aliquis habeat evidentiam de aliqua propositione et dubitet se habere evidentiam de illa propositione. Patet ex dictis satis," JEROME PARDO 1505, fol. 101va.

T). Also, note that the axiom of positive introspection is not cited explicitly.²³

As a counterargument to the second thesis, Pardo posits the scenario that someone knows that Socrates is running based on three pieces of evidence while doubting that such evidence is sufficient, and hence doubting that he truly knows that Socrates is running. Pardo elaborates on the set-up by emphasising that the agent knows that Socrates is running (presumably since the evidence is, in fact, sufficient) and even contemplates whether he knows that, but fails to know that he knows that Socrates is running.²⁴ The reason why Pardo ultimately dismisses the counterargument lies in the concept of evidence-based knowledge (*scire per evidentiam*): to doubt whether a statement is sufficiently supported by a piece of evidence is incompatible with the very notion of evidence. If evidence for a statement did not warrant it automatically, it would not be evidence in the first place.²⁵

23 A proof of the same thesis based on positive introspection might go as follows: if **p** doubts whether he knows that X is the case, then **p** fails to know that he knows that X is the case. However, if **p** fails to know that he knows that X is the case, then he does not know that X is the case (by the contraposition of positive introspection), and the original scenario states that **p** knows that X is the case. A contradiction.

24 "Sed circa secundam propositionem videtur esse dubium, nam videtur quod possibile est quod aliquis dubitet se scire Socratem currere. Nam pono casum quod tu scias Socratem currere per tres evidencias et dubites an ille tres evidencie sufficiant ad concludendum te scire Socratem currere et volo quod dubites an requirantur quattuor que non requirantur. Quo dato arguitur quod dubites te scire Socratem currere, quia tu scis Socratem currere et consideras sufficienter an scias Socratem currere et non scis te scire Socratem currere," JEROME PARDO 1505, fol. 101vb.

25 "Et cum dicitur quod aliquis potest dubitare an tres evidencie sufficiant ad sciendum vel an requirantur plures, respondeo: Si aliquis scit aliquam conclusionem per aliquam evidentiam, ita quod assensus eius qui est scientia causatur ex assensu premissarum qui dicitur evidentia, non potest dubitare an illa evidentia sufficit ad sciendum, quia hoc esset dubitare se scire. Qui enim scit per aliquam evidentiam, scit ita esse propter ita esse sicut significatur per talem evidentiam. Ideo dubitare an illa evidentia sufficiat est dubitare an ita sit assentiendum propter illud, quod includit opinari se scire," JEROME PARDO 1505, fol. 101vb.

2.2 Gaspar Lax: *Insolubilia* (1508/1512)

Gaspar Lax discusses iterated modalities as part of the fourth question of his *Insolubilia*.²⁶ The third article of this question asks whether someone can know and fail to know the same statement,²⁷ and includes *dubia*, the third of which is whether the same sentence can be (simultaneously and in the same sense) a matter of knowledge and doubt.²⁸ In this context, Lax discusses the argument that since someone can doubt whether he knows a certain sentence (to hold), he can also have knowledge and doubts regarding that sentence (even if the meaning of the sentence remains identical).²⁹ The argument splits into two parts: the proof of the inference and the proof of the antecedent, each of which is interesting in its own right.

The first sub-proof attempts to prove the validity of the aforementioned inference:

Proof of the inference. Since assuming that, I focus on that sentence and ask whether he firmly assents to that sentence or not. If he does, let that sentence be, for instance, this one: '*Socrates is running*'. Then I argue as follows: focusing on the inference '*Socrates is running and he firmly assents to this sentence which has such meaning; therefore, he knows that sentence*', the inference is valid and known to be valid by him, and he knows the antecedent, therefore, he knows the consequent, and as a consequence does not doubt that he knows that <sentence> in that sense.³⁰

26 "Utrum aliquo casu possibili posito stet eundem simul decipi et non decipi seu errare et non errare," GASPAR LAX 1512, f2vb.

27 GASPAR LAX 1512, f6ra.

28 GASPAR LAX 1512, g4vb.

29 "[S]tat aliquem dubitare se scire aliquam propositionem in aliquo certo sensu, ergo stat aliquem scire et dubitare eandem propositionem in eodem sensu," GASPAR LAX 1512, h1va.

30 "Consequentia probatur. Quia dato illo, capiam illam propositionem et queram: vel firmiter assentiet talis illi propositioni, vel ne. Si sic, sit illa gratia exempli ista: '*Socrates currit*'. Et sic arguo: capta ista consequentia: '*Socrates currit et ille firmiter assentit illi in tali sensu, ergo ille scit illa*', consequentia ista erit bona scita ab illo et ille sciet antecedens, ergo sciet et consequens, et per consequens non dubitabit istum scire illam in tali sensu," GASPAR LAX 1512, h1va.

The proof seems to have several problematic features. First, since the segment is meant to prove that if it is possible or self-consistent to doubt one's own knowledge, it is possible to know and doubt the same thing, so it should start with the former and end with the latter. However, the argument appears to start from the assumption that someone doubts whether he knows something and ends with proving that he knows that he knows it. That could easily be fixed by explicitly citing the rule that valid inferences are possibility-preserving, but such a move is missing.

Second, the question is asked whether the agent assents firmly to the sentence, '*Socrates is running*'. One might expect that both options will be discussed but only the positive one is, which seems illegitimate unless such an assumption is included in the scenario. That said, the argument could still count as a proof that a certain (sub-)scenario is self-consistent.

Third, after proving that the agent knows that he knows something, the argument proceeds to conclude that he does not doubt that he knows that, which is superfluous. Moreover, if the last step is correct, i.e., if the existence of knowledge entails the absence of doubt, then knowledge and doubt are mutually exclusive, which goes against the intention of the argument.

The sub-proof has some interesting features. An attempt to reach the epistemic iteration '*p knows that p knows that X is the case*' is made via the principle that knowledge distributes over implication or the axiom **K**, whereas the axiom of positive introspection or the axiom **4** is not endorsed explicitly. That may be on purpose, since the second sub-proof would undermine it, rendering the entire argument inconsistent. That said, a weaker form of introspection seems to be presupposed in the proof that the agent knows that the antecedent of the proposed inference holds, i.e., that Socrates is running and that he firmly assents to the sentence '*Socrates is running*'. The first part of the

antecedent is known as a result of the agent's firm assent to the sentence and the truth of the sentence, which qualifies as knowledge in a minimal sense.³¹ However, the second part of the antecedent can only be known if some principle along the lines of 'if p firmly assents to ϕ , p knows that p firmly assents to ϕ ' is presupposed, and that constitutes some form of introspection.

The second sub-proof introduces a scenario in which Socrates, based on observational data, firmly believes that the king is asleep, but doubts whether such evidence is sufficient.³² As a tacit assumption, the king is asleep, whence Socrates does actually know that the king is asleep as a result of having a firm, evidence-based and factually correct belief. In this scenario, Socrates doubts that he knows that the king is asleep; let us consider the inference: '*Socrates firmly assents to this sentence precisely for those reasons and Socrates knows the sentence (to hold), therefore those reasons sufficiently establish the knowledge of the sentence in question*'. The inference is held to be valid and known to be valid by Socrates, but its consequent is, by assumption, not firmly assented to by him, which means that he cannot firmly assent to the antecedent. However, Socrates is held to firmly assent to the first part of the antecedent. Therefore, Socrates does not firmly assent to knowing that the king is asleep, whence he does not know that he knows that, which implies that he doubts that he knows that the king is asleep.³³

31 For Lax's analysis of 'scire' and 'scire propositionem' see GASPARE LAX 1512, g1rb-g2ra.

32 "Iam probatur antecedens confirmationis: volo quod Socrates propter aliquas apparentias firmiter assentiat huic propositioni, scilicet '*rex dormit*', dubitet tamen an ille cause sint satis sufficientes ad hoc ut illa propositio sciatur in illo sensu et non habeat alias causas nec apparentias," GASPARE LAX 1512, h1va.

33 "Hoc dato sic arguo: ille dubitabit se scire illam propositionem in illo sensu, ergo propositum. Antecedens probo. Capió istam consequentiam: '*ille assentit firmiter solum propter illas causas illi propositioni et ille scit illam, ergo ille sunt sufficientes ad hoc quod talis propositio sciatur in tali sensu*'. Consequentia ista erit bona, scita a tali. Et ipse non firmiter assentiet consequenti, ut patet ex casu, ergo non firmiter assentiet antecedenti. Et firmiter assentit prime parti, ergo non firmiter assentiet secunde. Et per consequens non sciet se scire illam nec dissentiet tali propositioni, ergo dubitabit se scire illam," GASPARE LAX

Lax replies to this argument in two theses. The first is that someone can doubt whether he knows a certain non-self-referential sentence (to hold), proved by positing the scenario in which the agent is in doubt regarding his physical location, holding it possible that he is in Rome by a miracle, in which case he doubts whether, while being in Rome, he knows that the pope is asleep.³⁴ Such rather artificial scenarios aside, the second thesis states that the question of whether someone can be in doubt regarding his own knowledge is undecidable and that alternative plausible solutions to the sophism can be formulated.³⁵

2.3 Pardo, Lax and the Brito-Italian Tradition

Lax's and Pardo's texts have several interesting features. First, the criticism of the axiom of positive introspection takes two forms. The first emphasises the independence of first-order and second-order knowledge; a similar argument was introduced by John Wyclif and Paul of Venice.³⁶ The other employs the

1512, h1va. Note that the final step requires that the absence of knowledge implies doubt.

34 "Prima. Stat bene aliquem dubitare se scire aliquam certam propositionem non reflexivam in aliquo certo sensu. Propositio probatur, quia stat bene me nullo facto miraculo dubitare an sim Rome, dubitando an Deus fecerit aliquod miraculum, et ex consequenti me dubitare an sciam Rome illam: '*Papa dormit*' in illo sensu," GASPAR LAX 1512, h1vb.

35 "Secunda propositio. Sustenable est quod aliquis possit dubitare se scire aliquam certam propositionem non reflexivam in aliquo certo sensu non reflexivo, sciendo adhuc ipsum non esse nisi in illo loco. Et oppositum huius est sustenable. Nec potest evidenter aliquod istorum vel eius oppositum concludi. Et dicendo primo modo, diceretur ad illud quod tangis quod non staret illud te habente illos discursus, et proportionabiliter diceretur in aliis argumentis que possent contra hoc applicari. Dicendo secundo modo facile solveretur argumentum quod tangis dicendo quod non est possibile quod propter tales causas precise assentias tali et scias illam et dubites an ille sint sufficientes ad hoc quod scias illam," GASPAR LAX 1512, h1vb. Lax pays some further attention to scenarios involving self-referential phenomena and ultimately notes that a similar reasoning applies to other forms of iterated modalities, such as the problem of whether someone can doubt that he is in doubt (see GASPAR LAX 1512, h2ra).

36 See JOHN WYCLIF 1893, 184 (as Mark Thakkar kindly informed me, Dziewicki's edition of this passage is based on a particularly unreliable manuscript), analysed in HANKE

notion of evidence which allows that someone has a factually correct and evidence-based belief while being uncertain about the sufficiency of such evidence; similar arguments were employed by Peter of Mantua and Cajetan of Thiene³⁷ and Pardo's own reply echoes the approach of Cajetan of Thiene.³⁸ Second, the criticism of the axiom of positive introspection is introduced as part of proving the consistency of the scenario in which an agent doubts that he knows something. That is insufficient, but precisely what a scholastic author, knowledgeable upon epistemic logic, might be expected to do since it amounts to attacking a widely accepted strategy. Third, recall that when Pardo discusses the same scenario, he argues *only* against one particular threat to its consistency, arguably against the one which he views as the most reputable, which too can be interpreted as a historical contingency. These observations document that while Pardo and Lax came up with interesting innovations, they were to a significant degree perpetuators of the Brito-Italian logical tradition.

3. Epistemic Closure in *Posterior Analytics* Commentaries

In modern epistemic logic, the most elementary axiom, in the sense that the hierarchy of axiomatic systems is built up by adding further principles to it, is the principle that knowledge distributes over implication or the axiom **K**: if **p** knows that A implies B and **p** knows that A, then **p** knows that B.³⁹ In late-medieval logic, this principle plays multiple roles in various contexts: it is an

2018(1), 169–173, and PAUL OF VENICE 1499, fol. 81vb, analysed in HANKE 2018(2), 231–233.

37 See PETER OF MANTUA 1492, f7va (analysed in HANKE 2018(1), 174) and CAJETAN OF THIENE 1494, fol. 17ra (analysed in HANKE 2018(2), 234).

38 See CAJETAN OF THIENE 1494, fol. 17rb, analysed in HANKE, 2018(2), 241 (the same paper analyses the interesting contribution of Paul of Pergula).

39 See HUGHES, CRESSWELL 1996, 359–368. The same source can be consulted for the terminology of modern modal logic used throughout this paper.

inference rule in *consequentia* treatises (Ralph Strode), challenged in *insolubilia* treatises (Paul of Venice), and is used in setting up and solving sophisms in *de scire* treatises (William Heytesbury).⁴⁰ Here, a widely popular sophism whose precursors were introduced by Peter of Mantua and Paul of Venice will be discussed.⁴¹ The two Italians proposed similar sophisms targeting the notion of appearance, whose influence was acknowledged by the sixteenth-century authors. Peter of Mantua discussed the scenario in which two bodies are equidistant from an observer, to whom they appear equally long, namely one foot. If one of the bodies begins moving farther away from the observer to the point where it appears to be merely half a foot long, it would simultaneously appear to be half a foot long (by direct observation) and a foot long (by the earlier observation that the two bodies are equal).⁴² To outline Peter's solution, he makes two notes, each of which would solve the problem. First, he allows that the same object appears to be both a foot long and half a foot long based on different observational data (*secundum diversas apparentias*). Second, he introduces restrictions on the inferences containing the verb 'to appear' which invalidate the argument.⁴³

40 These examples are introduced in the aforementioned publications of Boh and Hanke.

41 The fact that the sophism is discussed by Peter of Mantua and Paul of Venice suggests that there could have been earlier British proponents of this sophism. While these were not mentioned in John Mair's circle, that might have been due simply to practical availability.

42 "Quarto. (...) per idem habetur in casu quod idem appareret tibi pedale et semipedale (...) Ponendo quod **a** et **b** pedalia distent equaliter a te que tibi appareant pedalia et equalia. Deinde incipiat removeri **b** per magnam distantiam et removeatur quousque apparebit tibi solum semipedale te bene sciente quod est pedale. Et sit iam medium instans hore in quo **b** appareat semipedale et arguitur quod **b** apparet tibi pedale et quod apparet semipedale. Quod enim **b** appareat tibi semipedale, apparet ex casu: **b** enim apparet sub duplo minori angulo quam ante apparebat et cetera sunt paria, igitur **b** apparet tibi minus quam ante apparebat. Sed quod **b** appareat tibi pedale arguitur, quia tu scis **b** esse pedale, quia scis **a** et **b** esse equalia, igitur credis **b** esse pedale. Et qualitercumque credis, taliter apparet tibi esse, igitur apparet tibi **b** esse pedale," PETER OF MANTUA 1492, g1vb.

43 "Ad quartum dicitur quod stat idem apparere pedale et semipedale secundum diversas apparentias. Negatur tamen in casu illo quod **b** appareat minus quam pedale. Et non

Paul of Venice's formulation of the problem is modified but similar; the sophism is part of the *de sensu composito et diviso* treatise of *Logica magna*.⁴⁴ His scenario assumes that there are three bodies named 'a', 'b' and 'c', such that **a** is imperceptibly longer than **b** and **b** is imperceptibly longer than **c**, but **a** is visibly longer than **c**. In that scenario, **a** and **b** appear to be equal, as do **b** and **c**, but **a** does not appear to be equal to **c**. However, given the apparent equality of **a** with **b** and of **b** with **c** and the transitivity of equality, **a** must appear to be equal to **c**: the inference '*a is equal to b and b is equal to c, therefore a is equal to c*' is known to be valid, therefore it appears to be valid, and the antecedent appears to hold, therefore, the consequent must appear to hold as well.⁴⁵ The argument proceeds in an unexpected way: rather than suggesting that apparent truth is preserved by inferences which are known to be valid, it insists that apparent truth (or validity) distributes over implication, which seems to be more controversial. Still, Paul's reply focuses on a different aspect of the argument and denies that the antecedent is apparently true, even though its parts are.⁴⁶

valet hoc argumentum: '*c, d et b apparent equalia, sed c apparet minus quam pedale, igitur b apparet minus quam pedale*', quia posito quod **c** esset unum minus quam pedale quod esset inter **a** et **b** in tanta distantia quod appareat semipedale, antecedens est verum et consequens falsum," PETER OF MANTUA 1492, g2ra.

44 *Logica magna* will be treated here as Paul of Venice's authentic work, such as it was viewed in John Mair's circle. That said, nothing important in this study rests on that assumption and the problem will not be discussed in detail. For the most recent discussion of this problem, see PAUL OF VENICE forth.

45 "Iuxta dicta solet dubitari persuadendo aliqua duo eidem apparere equalia et inequalia sic: ponatur a remotis **a, b, c**, et sit **a** insensibiliter maius **b**, sic quod appareat tibi **a** et **b** esse equalia, sit etiam **b** insensibiliter maius **c**, ita quod **b** et **c** appareant etiam tibi equalia, sed sit excessus **a** super **c** sensibilis, sic quod bene sentias **a** esse maius **c**. Isto posito patet quod **a** non apparet tibi equale **c**. Sed probatur quod sic. Et facio istam consequentiam: '*hoc a est equale huic b et hoc b est equale huic c, igitur hoc a est equale huic c*'. Ista consequentia apparet tibi bona, quia bene scis quod illa est bona, et antecedens apparet tibi verum, ergo et consequens," PAUL OF VENICE 1499, fol. 77va. In the only currently known manuscript of *Logica magna* (PAUL OF VENICE (ms.)), the corresponding passage is *Città del Vaticano*, Biblioteca Apostolica Vaticana, Vat.lat. 2132, fols. 98vb-99ra, but given its rarity, it is unlikely to have been the actual source.

46 "Ad primum respondetur negando quod antecedens illius consequentie apparet mihi

In John Mair's circle, Paul's sophism became a commonly discussed problem related to the axiom **K** and was referenced as having been introduced by Paul of Venice, with two notable adaptations. First, the sixteenth-century formulations replace 'appearance' with 'knowledge' and 'assent', while retaining the scenario and agreeing that equality is transitive or Euclidean and viewed as such. Second, while Paul of Venice was attributed with the reformulated version of the sophism, the references were not entirely precise. For instance, Antonio Coronel, who confirms the popularity of the sophism by labelling it as '*commune*', claimed that the problem was contained in the *de scire et dubitare* chapter of *Logica magna*.⁴⁷ Later on, Domingo de Soto would attribute the sophism to '*novi posterioristici*', probably referring to the contemporary Parisian logician, on top of mentioning Paul of Venice's '*sophisms de scire et dubitare*'.⁴⁸

2.1 John Mair: *In Petri Hyspani Summulas Commentaria* (1503/1505)

In addition to the chapters corresponding to Peter of Spain's treatises, John Mair's commentary on Peter of Spain's *Summulae* incorporated additional material, including *Liber Posteriorum*.⁴⁹ The adaptation of Paul of Venice's scenario is discussed in the first chapter of *Liber posteriorum*, which discusses the Aristotelian thesis that all knowledge and every doctrine comes about from pre-existing knowledge. To account for the difficult aspects of the theory,

verum, quia licet utraque pars illius antecedentis apparet mihi vera, non tamen apparet mihi quod utraque pars istius sit vera," PAUL OF VENICE 1499, fol. 77va.

47 See below for the details.

48 "Per haec aperitur via respondendi ad confirmationem, quae ex sophismatibus Pauli Veneti de scire et dubitare solent huc afferre novi posterioristici...", DOMINGO DE SOTO 1554, fol. 81rb). The passage is discussed in HANKE forth.(2), where the relation to John Mair's circle was not recognised.

49 Note that there is also an earlier 1503 separate edition of *Liber Posteriorum*, see JOHN MAIR 1503.

Mair discusses a series of *dubia*, the third of which addresses the thesis that the conclusion of a syllogism is cognised as soon as its premises are.⁵⁰ As that formulation appears too strong when applied to a human agent, Mair offers the following reformulation: as soon as an agent assents to both premises of a syllogism presented to him in their proper form, he assents to the conclusion, provided that he knows that the inference in question is valid and that all valid inferences which do not include any form of self-reference are truth-preserving; the same idea is, with a slight adaptation, restated in terms of knowledge rather than assent.⁵¹ There are two points of note. First, the rule to be challenged is formulated in terms of assent and knowledge, rather than apparent truth. Second, the complexity of the principle is reminiscent of similar approaches in the earlier scholastic tradition; note, in particular, the mentioning of self-referential phenomena, significant to John Mair's circle as an influence arising from Roger Swyneshed's suggestion that validity does not entail truth-preservation.⁵² The plurality of formulations bridges fourteenth- and fifteenth-century epistemic logic and sixteenth-century epistemology by giving two alternative expressions of what appears to be the intuitive core of

50 "Dubitatur de veritate huius propositionis: '*cognitis maiore et minore simul tempore cognoscitur conclusio*'," JOHN MAIR 1505, et5va.

51 "Ad tertium dubium, quod sic intelligitur quod quis potest maiori assentire antequam conclusioni assentiat et pari forma minori antequam conclusioni assentiat, sed quamprimum illis in forma positis assentit et scit consequentiam esse bonam, mox conclusioni assentit, dummodo sciat quod ex vero non sequitur propositio falsa non reflexiva," JOHN MAIR 1505, et6rb. This formulation is held equivalent to the following: "Si aliqua consequentia est bona, scita esse bona ab aliquo, si sciatur ita esse sicut significatur per antecedens, scitur ita esse sicut significatur per consequens, dummodo non repugnat consequenti sciri," JOHN MAIR 1505, et6rb-va. The second formulation is closer to the formulation typical of medieval treatises on consequences.

52 There is a growing body of research on this tradition. The fundamental editorial work includes ROURE 1962, SPADE 1979 and BRICOT 1986 and PAUL OF VENICE forth., and a number of recent and forthcoming publications by Stephen Read, including READ 2020(1) and READ 2020(2). For the general context of these approaches, see SPADE, READ 2018, for the philosophical context, see DUTILH NOVAES 2008. For John Mair's circle, see ASHWORTH 1974(2), 112-113 and ASHWORTH 1977; D'ORS 1986; HANKE 2012, 2013 and 2014. For the role of paradoxes in scholastic epistemic logic, see (among others) HANKE forth.(1).

the axiom **K**.

As one of the counter-examples to the principle, Mair restates Paul of Venice's scenario: let us posit that there are three bodies possessing the physical properties described by Paul of Venice. The scenario is held to be possible (i.e., acceptable for the sake of argument). Now let us assume that the following inference is proposed to Socrates: '*things that are equal to the same thing are also equal to one another, a and c are equal to the same thing (namely to b); therefore, a and c are mutually equal*'. In this scenario, Socrates is assumed to know that the inference is an instance of DARII, i.e., a valid syllogistic inference. The first of the premises is a 'common notion' (*communis animi conceptio*), whence Socrates must assent to it and the second premise is assented to by hypothesis. However, Socrates would not assent to the consequent (based on perceptual evidence). As a result, the scenario is a counter-example to the axiom **K** (in one form or another).⁵³

Mair provides a two-step reply. First, he considers the argument irrelevant (to the context of *Posterior Analytics*), since the second premise is not cognised in a sense relevant to the Aristotelian notion of scientific knowledge or demonstration. Second, he denies that Socrates would assent to both premises in this scenario: if Socrates knew that the consequent of the aforementioned inference is false (based on perceptual evidence), that validity entails truth-preservation (in all relevant cases) and that the first premise is true, he would *immediately* dissent to the second premise, and his intellect would re-e-

⁵³ "Contra regulam (...) instatur. Bene sequitur: '*quecumque sunt equalia uni tertio sunt equalia inter se, a et c sunt equalia uni tertio (scilicet b), ergo a et c sunt equalia*'. Stat quod aliquis assentiat antecedenti et tamen non assentiat consequenti, esto quod sciat consequentiam esse bonam, igitur minor patet. Posito quod **a** et **b** sint duo corpora ferme equalia, sed **b** **a** exuperet imperceptibiliter et **c** **b** excedat etiam imperceptibiliter, sed **c** **a** sensim excedat. Totus casus est possibilis, maior est communis animi conceptio, ergo assentit illi Socrates, et assentit minori per ypotesim, igitur toti antecedenti et non assentit consequenti, et pono quod sciat consequentiam esse in DARII," JOHN MAIR 1505, et6va.

valuate the perceptual evidence as flawed.⁵⁴

2.2 Antonio Coronel: *Commentaria in Posteriora Aristotelis* (1510/1528)

Antonio Coronel came up with what is fundamentally the same scenario to challenge the same principle, but developed it in a different way. The agent in the scenario is assumed to have assented to the premises of: “*things that are equal to the same thing are also equal to each other, a and c are equal to the same thing (namely to b); therefore, a and c are equal to each other,*” after which he would form that inference. In that case, the agent would assent to both premises and the inference evident to him, but not to the conclusion. The reasons are the posited visual counter-evidence and the fact that nobody can both assent to and dissent from the same statement at the same time.⁵⁵

Coronel presents three possible solutions. The first two are attributed to Paul of Venice and Cajetan of Thiene, the unattributed third is refined into

54 “Respondetur: quicquid sit argumentum, non est contra mentem Aristotelis, quia talis non cognoscit minorem, capiendo cognoscere sicut Aristoteles capit. Sed ad argumentum in se, quia is non assentit antecedenti totali, immo dissentit minori, postquam videt consequens esse falsum, et ad sensum hoc percipit, et scit quod ex vero non sequitur falsum (saltem extra reflexivas) et scit maiorem esse veram, statim minori per intellectum dissentit, quia sensum delirantem ille castigat,” JOHN MAIR 1505, et6va.

55 “Ad secundum dubium in quo queritur utrum cognita maiore et minore simul tempore cognoscatur conclusio. (...) Primo arguitur in aliquo casu stat assentire antecedenti et consequentiae et non conclusioni, igitur conclusio falsa. Antecedens probatur. Ponatur casus communis: sint tria corpora coram te **a**, **b**, **c**, **a** insensibiliter maius **b**, taliter quod per sensum non possis iudicare **a** esse maius **b**, sed precise quod est ei equale, et **b** insensibiliter maius **c** propter eandem causam, sed **a** sit sensibiliter maius **c**, taliter quod sensu percipere potes **a** esse maius **c**. Volo quod per totam horam preteritam tu assentiebas isti copulative: ‘*quecumque sunt equalia uni tertio sunt equalia inter se, a et c sunt equalia uni tertio, scilicet b*’. Et in hoc instanti primo formes completam consequentiam, sic dicendo: ‘*ergo a et c sunt equalia inter se*’. Et arguitur sic: In hoc instanti habes assensum maioris et minoris et consequentiae (cum sit tibi evidens). Et non assentis conclusioni, quod patet: dissentis ei, ergo non assentis ei, cum non possis eidem propositioni simul et semel assentire et dissentire. Antecedens probatur, nam sensu percipis **a** esse maius **c**, ergo propositum. Hoc argumentum est commune in hac materia,” CORONEL 1510, fol. 4rb–va.

Coronel's own solution. Needless to say, the correspondence between the actual sources and Coronel's presentation is rather loose. Since this is common to other authors from John Mair's circle, there appears to be a common source to this (mis)conception.

First, Paul of Venice is claimed to have admitted the scenario as possible while denying that the agent assents to the antecedent of the aforementioned inference, even though he assents to each of its parts. The assent to the entire antecedent is held to be different from the two partial assents. This position is referenced back to Paul's *Logica, capitulo de scire et dubitare*,⁵⁶ which is incorrect for several reasons. First, the scenario is discussed in a chapter other than that indicated. Second, Coronel shifts from apparent truth to assent, which changes the principles at stake. Third, the theory of mental acts seems imposed on Paul's text.

Coronel rejects the solution by suggesting that such an agent would surely agree that conjunction introduction is legitimate, which should guarantee the existence of the required act of assent. In other words, assent can be assumed to be closed under conjunction introduction, which Coronel supports with two reasons. First, the opposite assumption postulates an agent with an unrealistic degree of logical incompetence and, as Longeway noted, ultimately results in an infinite regress.⁵⁷ Second, as an interesting move from pure logic to psychology, the assent to the conjunctions is assumed to be generated *causally* by the pre-existing assents to the sub-formulas⁵⁸.

56 "Ad hoc respondet Paulus in sua *Logica*, capitulo de scire et dubitare casu admisso, negando quod assentias antecedenti, licet assentias maiori et minori. Assensus totius antecedentis est distinctus ab assensu partium et tu non habes illum tertium assensum," CORONEL 1510, fol. 4va.

57 LONGEWAY 2009, 401–402.

58 "Hec solutio nulla est, nam bene sequitur: quelibet pars huius copulative est vera, ergo tota copulativa est vera. Sed iste stante toto casu, si formaret istam consequentiam, assentiret ei et antecedenti eius, ergo assentiret consequenti. Ad hoc argumentum respondet ipse, quod ille talis non assentiret consequentie. Sed hoc nichil est, nam suppono

The second strategy is attributed to Cajetan of Thiene's commentary on Heytesbury's *De scire et dubitare*. According to Coronel's formulation, the assents to different parts of the inference are performed by different cognitive powers: intellect assents to the underlying mathematical axiom while senses assent to two bodies being equal to a third body. For that reason, someone can assent to both premises and to the inference but not to the conclusion.⁵⁹ It is not clear to which passage Coronel is referring: while certain parts of Cajetan's commentary on Heytesbury use the distinction between perceptual and intellectual, that appears to be limited to different modes of reference to an object within specific scenarios.⁶⁰

Coronel's reply is distorted in printed editions of the text, but its content is relatively comprehensible: the distinction does not solve the problem, since intellect follows senses in the absence of a counter-argument, and the entire argument could be restated for an angel or a soul separated from the body. Coronel does not elaborate on the second point and it is not clear how such reformulations should work, when the argument relies on *perceptual* distinguishability.⁶¹

The third position naturalises the issue and the discussion of the axiom **K** turns into a debate on whether cognitive changes are instantaneous or suc-

quod sciat aliquid in logica et ex consequenti quod sciat ad veritatem copulative sufficere cuiuslibet partis veritas. (...) Tum tertio: ille tertius assensus, si ponendus est, causatur ab assensibus partium copulative. Sed iste per totam horam preteritam habuit assensum partium copulative, ergo habuit illum assensum," CORONEL 1510, fol. 4va.

59 "Ideo ponitur alia solutio, que est Gaethani de Thienis in *Commento de scire et dubitare Hentisberi*, quod in illo casu Socrates assentit antecedenti, sed non per eandem potentiam: assentit maiori per intellectum et non per sensum, minori opposito modo. Unde non est inconveniens assentire maiori et minori et consequentie et non conclusioni, quando non assentitur maiori et minori per eandem potentiam," CORONEL 1510, fol. 4va.

60 See CAJETAN OF THIENE 1494, fols. 19ra and 19va.

61 "Hoc solutio nulla est, nam cum primo sensus assentit minori, intellectus ei assentit, cum non habeat rationem in oppositum. Tum tertio, quia argumentum potest fieri de uno angelo vel de anima separata," CORONEL 1510, fol. 4va. Cf. CORONEL 1528, fol. 5va.

cessive, how to analyse beginning and ceasing, the nature of the causal potency of certain cognitive acts, and so on, starting with the thesis that as soon as Socrates assents to the inference and dissents from the conclusion, the assent to the second premise ceases to exist.⁶² Such naturalisation of logic is coherent with other issues discussed by Coronel, such as quantitative limitations upon the capacity of the human mind (which could be a sign of continuity with the Oxford Calculators).⁶³

2.3 Juan de Celaya: *Expositio in libros Posteriorum Aristotelis* (1517/1521)

Juan de Celaya addresses Paul of Venice's scenario while discussing the validity of the axiom **K** applied to syllogistic inferences.⁶⁴ The scenario is formulated for Socrates playing the role of the agent.⁶⁵ The most significant difference compared with Mair and Coronel is that the inference proposed to Socrates is simply '*a is equal to b and b is equal to c; therefore, a and c are mutually equal*'. In such a scenario, Socrates is held to assent to both premises based on empirical evidence and to the proposed (syllogistic) inference, since he is a

62 The series of propositions and counter-arguments starts with the following statement: "Ideo ponitur tertia solutio que talis est: in primo instanti in quo Socrates assentit consequentie et dissentit conclusioni per primum non esse corrumpitur assensus minoris. Tunc dicitur ad formam argumenti cum sic arguitur: Socrates in hoc instanti assentit maiori et minori etc., nego istam, quia hoc instans est primum non esse assensus minoris," CORONEL 1510, fol. 4va. For the details of Coronel's position, see LONGEWAY 2009, 403–406.

63 See LONGEWAY 2009, 406–418. For a similar discussion in Soto's commentary on *Posterior Analytics*, see HANKE forth.(2).

64 "Queritur circa hunc textum an illa secunda conclusio Philosophi, scilicet cognitis maiore et minore simul tempore cognoscitur conclusio sit vera. (...) Quarta conclusio est ista: impossibile est cognoscere maiorem et minorem et bonitatem consequentie ad sensum declaratum quin in eodem instanti cognoscatur conclusio," JUAN DE CELAYA 1517, fols. 16vb and 17ra. There is a second edition, issues 1521, see JUAN DE CELAYA 1521.

65 "Secundo principaliter arguitur: possibile est Socratem assentire maiori et minori et bonitati consequentie non assentiendo conclusioni, igitur illa conclusio falsa. Antecedens probatur: volo quod sint tria corpora ante Socratem, scilicet **a**, **b** et **c**, **a** sit imperceptibiliter maius **b**, similiter **b** sit imperceptibiliter maius **c**, **a** vero sit perceptibiliter maius **c**," JUAN DE CELAYA 1517, fol. 17va.

competent logician – a point repeated several times.⁶⁶

Celaya presents three solutions to the problem, the first two of which are attributed to Paul of Venice and Cajetan of Thiene. Celaya's presentation of Paul of Venice's solution to the problem does not differ from Coronel's in any significant way, except for the fact that the reference is correct.⁶⁷ The suggestion that Socrates would assent to each premise separately but not to their conjunction is dismissed by pointing out its inconsistency with Aristotelian epistemology and by insisting that Socrates is assumed to be a competent logician who is paying attention to the problem.⁶⁸ Overall, this seems to be Coronel's reply, dressed up in different clothes. Similarly, Celaya's presentation and criticism of Cajetan's position is identical to Coronel's (minus the

66 "Et proponatur Socrati iste sillogismus: '*a est equale b et b est equale c, ergo a et c sint equalia*'. Tunc Socrates assentiet maiori et minori et bonitati consequentie et non assentiet conclusioni, quia habet scientiam de eius contradictorio, igitur antecedens verum. Quod Socrates assentiet maiori et minori patet, quia ad experientiam apparent sibi vere maior et minor, cum non possit distinguere seu discernere excessum **a** corporis supra **b** corpus nec excessum **b** corporis supra **c** corpus. Quod Socrates assentiet bonitati consequentie probatur: suppono quod illi tres termini **a, b, c** sint termini discreti significantes illa tria corpora. Socrates est bonus logicus et advertit circa bonitatem illius consequentie, ut suppono, et illa consequentia est bonus sillogismus expository, ergo assentit illi," JUAN DE CELAYA 1517, fol. 17va.

67 "Ad hoc argumentum respondet Paulus Venetus in prima parte *Logice magne*, capite de *sensu composito et diviso*, concedendo antecedens et negando consequentiam. Ratio assignatur ab eo, quia licet Socrates assentiat maiori et minori, non tamen assentit toti antecedenti. Nam non habet Socrates in illo casu unum assensum circa totum antecedens, sed duos," JUAN DE CELAYA 1517, fol. 17vb.

68 "Hec solutio parum aut nihil valet, nam obviat Philosopho, qui oppositum asserit. Item si Socrates est bonus logicus, considerat circa copulativam, que est antecedens, et assentit cuilibet parti copulative, ergo assentiet toti copulative, cum sciat bene ad veritatem copulative sufficere utramque partem principalem esse veram. Et per consequens habebitur quod Socrates assentit toti antecedenti et bonitati consequentie et non conclusioni, quod est contra Philosophum et ipsummet Paulum Venetum," JUAN DE CELAYA 1517, fols. 17vb. The emphasis on attention may be significant: the argument assumes that Socrates agrees to two statements and pays attention to whether their conjunction holds and is familiar with the rule of conjunction introductions. In general, this amounts to the axiom **K** extended by the requirement of attention, and such extensions are characteristic of Italian scholasticism (see HANKE forth.(1)).

flaws).⁶⁹ These similarities suggest that the two commentaries are not independent.

The third solution develops the scenario to account for possible objections. As the first step, Celaya states that Socrates could assent to both premises without assenting to the inference '*a is equal to b and b is equal to c; therefore, a and c are mutually equal*'.⁷⁰ Second, if the scenario is modified by additionally positing that Socrates contemplates whether the inference is valid, it follows that he would assent to its validity, which would immediately prevent him from assenting to both premises as a result of his logical competence.⁷¹ The strategy can be restated even if one assumes that belief-changes such as ceasing to assent are successive processes that do not permit an instantaneous change.⁷²

69 "Ideo aliter respondet Gaietanus de Tyennis, commentator Hentisberi, concedendo quod in illo casu Socrates assentit maiori et minori et bonitati consequentie et etiam toti antecedenti et non conclusioni, sed non per eandem potentiam, sed per diversas. Nam per potentiam sensitivam assentit antecedenti et per potentiam intellectivam assentit bonitati consequentie. Nec hoc est contra Philosophum, quia Philosophus intelligit per unam et eandem potentiam. Ista solutio etiam non valet. Nam quamprimum sensus assentit alicui propositioni, intellectus assentit eidem, nisi habeat aliquam rationem fortem in oppositum. Insuper argumentum potest deduci de anima separata vel de angelo," JUAN DE CELAYA 1517, fol. 17vb.

70 "Propter hoc igitur aliter est respondendum ad argumentum. Unde ad formam argumenti negatur antecedens et ad probationem admissio casu, concedo quod assentiat maiori et minori, nego tamen quod assentit bonitati consequentie," JUAN DE CELAYA 1517, fol. 17vb.

71 "Et ad probationem, si ponas quod advertat circa bonitatem consequentie, concedo quod assentiet bonitati consequentie, nego tamen quod tunc assentiet maiori et minori, immo quamprimum assentit bonitati consequentie, dissentiet alicui premissarum, postquam est bonus logicus. Nam videbit clarissime bonitatem consequentie et falsitatem consequentis, eliciet ergo statim falsitatem antecedentis. Et per consequens necessario dissentiet tunc alicui premissarum, et sic numquam habebuntur illi tres assensus sine assensu conclusionis," JUAN DE CELAYA 1517, fol. 17vb.

72 "Et si supponas quod quilibet assensus debeat corrumpi successive, facile est etiam respondere, nam numquam habebitur in illo casu assensus bonitatis consequentie, donec aliquis assensus premissarum fuerit destructus. Et si dicas: quid impedit Socratem habere assensum bonitatis consequentie, postquam est bonus logicus et advertit circa illam, ad hoc dico quod dissensus conclusionis impedit, nam Socrates ideo quia est bonus logicus videt falsitatem consequentis et credit antecedens esse verum, ea de re dubitat

Third, Celaya introduces the following objection. Assume that Socrates, who is a competent logician, regards a certain conclusion as conjectural, and then a proof of the conclusion is presented to him, which he contemplates. Afterwards, Socrates starts contemplating the syllogistic premises of the proof that are assumed to have the same degree of apparency (*apparentia*). Based upon these assumptions, Socrates would have to assent to both premises: he must assent to one of them, but he cannot favour either of them, since they are equally evident. He is assumed to have assented to the inference. However, he would not assent to the conclusion (since knowledge and conjecture are mutually exclusive), which is regarded as undesirable or simply false.⁷³

Celaya denies that Socrates would assent to both premises in this scenario and offers two different strategies without indicating a clear preference for either. The first turns the original argument around: the parity of evidence may be what prevents Socrates' assent to either premise. The second suggests that Socrates could assent to one of the premises simply as a matter of a deliberate (rather than rationally warranted) decision.⁷⁴

de bonitate illius consequentie," JUAN DE CELAYA 1517, fol. 17vb.

73 "Contra istam solutionem arguitur. Sequeretur quod si Socrates haberet opinionem alicuius conclusionis et adducatur illi demonstratio illam demonstrans et advertat Socrates immediate circa bonitatem consequentie et sit bonus logicus et postea advertat circa maiorem et minorem simul et semel et habeat tantam apparentiam adequate erga unam premissam sicut erga aliam, tunc Socrates assentiet bonitati consequentie et maiori et minori et non assentiet conclusioni. Consequens est falsum, igitur. Quod assentiet bonitati consequentie non est dubium. Quod assentiat maiori et minori probatur: alicui illarum assentiet postquam habet motivum. Et non est maior ratio de una quam de alia, postquam habet equalem apparentiam erga ambas, ergo ambabus assentiet. Quod non assentiet conclusioni probatur: dato opposito sequitur quod habebit opinionem et scientiam de eadem conclusione, quod est impossibile, igitur nullo modo assentiet conclusioni," JUAN DE CELAYA 1517, fols. 17vb–18ra.

74 "Ad hanc replicam respondetur negando sequelam pro illa parte, scilicet quod assentiet maiori et minori. Et ad probationem: Potest negari quod assentiet alicui illarum premissarum, postquam ex casu habet tantam apparentiam erga unam sicut erga alteram. Potest etiam concedi quod alicui illarum assentiet. Et negatur quod non sit maior ratio de una quam de alia. Et pro ratione debet assignari libertas voluntatis. Nam postquam ha-

4. Bradwardinian Semantics and Epistemic Sophisms

As part of developing two epistemic scenarios in *Regule solvendi sophismata*, Heytesbury confronts the Bradwardinian idea that sentential meaning is closed under entailment.⁷⁵ Since he treats knowledge as fundamentally linguistic by typically using phrases such as ‘*p* knows the sentence ξ ’, the idea that sentential meaning is closed under entailment has significant implications for his logic.⁷⁶ In John Mair’s circle, that problem appears to be relatively rare and only one representative thereof can be introduced at this point. This could obviously mean that some sources have been omitted and will resurface in future. However, there is an argument to be made why such debate could be genuinely rare in John Mair’s circle. The idea that sentential meaning is closed under entailment was for Heytesbury tied to the treatment of semantic paradoxes. As opposed to the circle of the Oxford Calculators, where that idea was coined by Bradwardine and Heytesbury, Mair’s students were more likely to develop the treatment of paradoxes introduced by Roger Swyneshed, which implies a critical stance towards Bradwardinian seman-

bet equalem apparentiam circa illas, voluntas pro libito suo potest se determinare ad partem quam voluerit, utputa ad maiorem vel minorem,” JUAN DE CELAYA 1517, fols. 18ra. Through the ‘*non est maior ratio*’ part, this passage is tied to the analysis of the no-no paradox; for a philosophical analysis of the problem based on the scholastic material, see READ 2006, for Mair’s discussion of the paradox, see HANKE 2012, 167–168.

⁷⁵ See THOMAS BRADWARDINE 2010, which replaces the earlier edition in ROURE 1970. Furthermore, an interesting debate of Bradwardinian semantics is published in RAHMAN, TULENHEIMO, GENOT 2008. For Heytesbury’s discussion as related to epistemic sophisms, see HANKE 2021 and HANKE forth.(1), where further references are discussed (together with the positions of Heytesbury and Paul of Venice). The view that sentential meaning is closed under entailment develops the treatment of semantic paradoxes discussed in the first chapter of *Regule solvendi sophismata* (which is, together with other sources, discussed in PIRONET 2008). As suggested by Yrjönsuuri, Heytesbury’s position can be explained by the influence of Thomas Bradwardine: see YRJÖNSUURI 2008, 599–600.

⁷⁶ The idea that linguistic objects, specifically sentences, are the proper subject-matter of knowledge was in the contemporary context famously introduced by William of Ockham (see SPADE, PANACCIO 2019 referencing further sources). To what extent Heytesbury is developing the same notion of knowledge is not clear: his choice might have been more directly influenced by the genre of *obligationes* in which the problem is presented.

tics.⁷⁷ The one currently known exception is Mair’s predecessor Jerome Pardo who introduced the problem as part of the seventh counterexample to the principle that the same proposition cannot be known and conjectured at the same time. The sophism he discusses can be traced back to Heytesbury and Paul of Venice. To show his probable sources, let us consider four formulations of the problem offered by Heytesbury in *Regule solvendi sophismata*, by Paul of Venice in his *Logica magna* and *Sophismata*, and by Pardo in his *Medulla dialectice*.⁷⁸ The scenario consists in positing that an agent knows that someone is either Socrates or Plato, while doubting or not knowing which. The comparative analysis suggests that there are two different formulations of the problem, the first proposed by Heytesbury and by Paul of Venice in *Sophismata*, the other by Paul of Venice in *Logica magna* and by Pardo:

<i>Regule solvendi sophimata</i> ⁷⁹	<i>Sophismata</i> ⁸⁰	<i>Logica magna</i> ⁸¹	<i>Medulla dialectice</i> ⁸²
[1a] Item posito quod scias quod hoc sit Socrates vel Plato, nescias tu tamen an hoc sit Socrates nec scias an hoc sit Plato.	[1b] Quarto pono quod tu scias hoc esse Socratem vel Platonem, dubites tamen, an sit Socrates, et dubites, an sit Pla-	[1c] Tertio arguitur sic. Et pono quod hoc sit Sortes, quod scias esse Sortem vel Platonem, sed lateate an sit Sortes	[1d] Septima instantia. Hec est scita a te ‘hoc est Socrates’ et dubia, ergo contra conclusionem.

77 See above for the relevant literature (fn. 52).

78 The fourteenth-century debate extends beyond these examples, but these appear sufficient for the present purposes and were available in printed editions. To mention another important example, the slightly modified sophism is discussed in JOHN HUNTER 1999, 435. This is noteworthy, since Hunter appears to be one of *Logica magna*’s sources, as argued by Hughes in his edition of PAUL OF VENICE 1990, *ad indicem*.

79 WILLIAM HEYTESBURY 1494, fol. 12vb (emphasis mine).

80 PAUL OF VENICE 1493, fol. 51rb (emphasis mine).

81 PAUL OF VENICE 1981, 86 (the style of the quotation was modified to enhance coherence with other quotations, emphasis mine).

82 JEROME PARDO 1505, fol. 101rb (emphasis mine).

	to.	et lateat te an sit Plato.	
		Isto posito haec propositio ' <i>hoc est Sortes</i> ' est scita a te et eadem est tibi dubia, ergo scitum a te est tibi dubium. Patet consequentia et antecedens probatur:	Et pono quod dubites an hoc sit Socrates vel hoc sit Plato, scias tamen hoc esse Socratem vel Platonem (uno illorum demonstrato).
[2a] Et tunc erit ista propositio tibi dubia: ' <i>hoc est Socrates</i> '.	[2b] Et patet quod ista propositio: ' <i>hoc est Socrates</i> ' est tibi dubia.	[2c] Nam quod illa sit tibi dubia patet per casum.	[2d] Quod illa: ' <i>hoc est Socrates</i> ' sit tibi dubia patet ex casu.
[3a] Et quod ista sit scita a te, arguitur sic, quia tu scis illam significare precise sicut tu scis esse. Quod probo, quia tu scis quod illa significat quod hoc est Socrates vel hoc est Plato et tu scis ita esse	[3b] Sed arguitur, quod est scita a te, quia tu scis hoc esse Socratem vel Platonem, et scis quod ista sic adequate significat, ergo tu	[3c] Et quod illa sit scita a te probatur: Tu scis eius significatum primum, quod scis significari primarie per illam, ergo tu scis illam. Antecedens probatur: tu scis hoc esse Sortem vel Platonem, et hoc est significatum primum illius, quod scis	[3d] Sed quod sit scita, probatur, quia tu scis hoc esse Socratem vel Platonem et illa propositio significat hoc esse Socratem

<p>(scis enim quod hoc est Socrates vel quod hoc est Plato), igitur tu scis istam propositionem.</p> <p>Consequentia patet. Minor etiam patet ex casu.</p>	<p>scis istam.</p> <p>Patet consequentia cum maiori.</p>	<p>esse suum significatum primarium, igitur etc.</p> <p>Patet consequentia cum maiori per casum.</p>	<p>vel Platonem, ergo illa propositio est a te scita.</p>
<p>[4a] Et maior arguitur, quia tu scis quod illa precise significat quod hoc est Socrates et scis quod sequitur: <i>'illa significat precise quod hoc est Socrates, ergo illa significat precise quod hoc est Socrates vel quod hoc est Plato'</i>.⁸³</p>	<p>[4b] Et minorem declaro, nam tu scis istam: 'hoc est Socrates' significare adequate hoc esse Socratem, ergo tu scis istam significare adequate hoc esse Socratem vel Platonem.</p>	<p>[4c] Et minor probatur, nam haec propositio: 'hoc est Sortes' significat primarie hoc esse Sortem, ergo illa significat primarie hoc esse Sortem vel Platonem.</p>	<p>[4d] Minor probatur, quia significat hoc esse Socratem, ergo significat hoc esse Socratem vel Platonem.</p>

⁸³ Some copies of Heytesbury's texts have preserved a different, shorter version of the argument, which is even closer to *Logica magna*; as an example, see the following passage from one Leipzig manuscript: "Maior arguitur sic: tu scis quod ista: *'hoc est Socrates'* precise significat quod hoc est Socrates per casum, ergo tu scis quod ista precise significat quod hoc est Socrates vel quod hoc est Plato. Probatur consequentia: arguitur enim a parte disiuncti ad totum disiunctum sine negacione et sine distribucione," WILLIAM HEYTESBURY (ms.), Leipzig, Universitätsbibliothek, ms. 1360, fol. 112va (this diversity of *Regule's* textual tradition is discussed in HANKE forth.(1)). Note that both versions of the argument are described in the same way, but the 'short version' appears to be a closer fit to the description, as the 'long version' would require a more detailed description. Other examples can be disregarded in this study, but if Paul of Venice used Heytesbury's treatise as a direct source in this passage, he appears to have used the shorter version of the argument. However, John Hunter's version is close to the 'short version' too: "Tu scis istam significare primo hoc esse Johannem. Ergo tu scis istam significare primo hoc esse Johannem, vel Platonem," JOHN HUNTER 1999, 435.

<p>Arguitur enim a parte disiuncti vel a parte disiunctive ad totam disiunctivam sine negatione et sine distributione. Ergo etc.</p>	<p>Patet consequentia a parte disiuncti ad totum disiunctum, antecedens est concedendum a te, ergo et consequens.</p>
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<p>Patet consequentia a parte disiuncti ad totum disiunctum, et antecedens est verum, igitur et consequens.</p>	<p>Patet consequentia a parte disiuncti ad totum disiunctum.</p>
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All four arguments posit the scenario in which the agent knows that some person is Socrates or Plato, but is uncertain whether that person is Socrates and is uncertain whether that person is Plato. The key doctrinal difference between the different formulations is in rows [3] and [4]. Note that while the same rule (disjunction-introduction) is cited in [4a]–[4d], it applies to different moves. All four texts attempt to prove that the agent knows the sentence ‘*this is Socrates*’ in the posited scenario. In all four versions of the argument ([3a]–[3d]), the agent knows that the person in question is Socrates or Plato. In [3a], [3b] and [3c], the knowledge of ‘*this is Socrates*’, is derived from the knowledge that the person is Socrates or Plato and the knowledge that the sentence ‘*this is Socrates*’ signifies that the person is Socrates or Plato (which is true). As opposed to that, Pardo’s argument [3d] only operates with the knowledge that the person is Socrates or Plato and the fact that that is what the sentence ‘*this is Socrates*’ signifies, which could be due to weaker requirements on sentential knowledge.⁸⁴ More significantly, there is a split between [4a] and [4b] vs [4c] and [4d] in the supporting argument: while [4a] and [4b] attempt to prove that the agent knows that the sentence ‘*this is Socrates*’ signifies that the person in question is Socrates or Plato, [4c] and [4d] only attempt

⁸⁴ Another option is that something is missing from Pardo’s text, but the two editions are in agreement on this point.

to prove that this is what the sentence signifies. In other words, the key inferences are as follows:

[4a/4b] You know that '*this is Socrates*' signifies precisely/adequately that this is Socrates; therefore, you know that '*this is Socrates*' signifies precisely/adequately that this is Socrates or Plato.

[4c/4d] '*This is Socrates*' signifies (primarily) that this is Socrates; therefore, '*this is Socrates*' signifies (primarily) that this is Socrates or Plato.

For [4b], this move is not further justified, while [4a] assumes that the agent under scrutiny knows that an inference along the lines of [4c/4d] is valid, which allows the reduction of [4a] to an instance of the axiom **K**.⁸⁵ Either way, the passage includes sentences with considerably complicated logical structure, including the concatenation of two hyperintensional operators, which alone makes the argument open to the objection that it is not a straightforward instance of disjunction-introduction.⁸⁶

The original solution consisted in formulating certain principles of epistemic logic that would be sensitive to different forms of sentential meaning; ultimately, Heytesbury denied that the agent is aware of '*this is Socrates*', signifying that this is Socrates or Plato and even that this is the sentence's primary and principal signification.⁸⁷ The underlying reason seems to be that it is unrealistic to require that the agent is aware of the full meaning of a sentence if sentential meaning is closed under entailment. Two straightforward solutions to this problem are to propose a more realistic requirement, such as its restriction to 'primary' or 'explicit' meaning, or to abandon the view that the

⁸⁵ The aforementioned 'short version' is identical to [4b] in this regard.

⁸⁶ For the notion of hyperintensionality, see CRESSWELL 1975.

⁸⁷ "Ad quartum argumentum admitto totum usque ad hoc quod dicitur quod scio quod hec propositio '*hoc est Socrates*' significat precise quod hoc est Socrates vel quod hoc est Plato. Illam nego, nec illa primo et principaliter sic precise significat, sed primo et principaliter illa significat quod hoc est Socrates," WILLIAM HEYTESBURY 1494, fol. 15rb.

subject matter of propositional attitudes are sentences; as Heytesbury is (for whatever reason) unwilling to do the latter, he is prone to doing the former. The same part of the argument is attacked by Paul of Venice in *Sophismata*, with the exception that Paul rejects it in terms of a terminist analysis of hyperintensional contexts generated by the combination of the term ‘adequate’ with a hyperintensional verb ‘significat’.⁸⁸

In the *Logica magna* version of the argument, Paul of Venice denies that *primary* meaning is closed under entailment and notes that the inference in question is not an instance of disjunction-introduction by virtue of the presence of a modal operator.⁸⁹ This strategy is identical to the one endorsed in *Sophismata*, with the exception that it applies to a less complicated sentential context. Similar to Heytesbury, Paul of Venice continues by asking whether sentential meaning is closed under entailment; while he denies such closure to *primary* meaning generated by the terms of the sentence in this context, he is (surprisingly) open to that idea in general.⁹⁰

88 “Ad quartum respondetur consimiliter negando istam: ‘tu scis hanc: “hoc est Socrates” significare adequate hoc esse Socratem vel Platonem’. Et ad probationem: negatur consequentia, quia arguitur ad disiunctum stans collective ratione illius termini ‘adequate’ et termini concernentis actum mentis, sicut non sequitur: ‘iste terminus “homo” primo significat hominem, igitur significat primo hominem vel asinum’,” PAUL OF VENICE 1493, fol. 51vb.

89 “Ad tertium argumentum respondetur concedendo quod illa ‘Hoc est Sortes’ est mihi dubia. Et cum probatur quod illa est scita a me, quia scio hoc esse Sortem vel Platonem et illa significat primarie hoc esse Sortem vel Platonem, nego quod illa significat primarie hoc esse Sortem vel Platonem, sed significat primarie hoc esse Sortem, ex quo non sequitur quod significat primarie hoc esse Sortem vel Platonem. Et si arguitur: ‘...a parte disiuncti ad totum disiunctum’, respondeatur ut prius in proximo argumento,” PAUL OF VENICE 1981, 94. The passage he seems to refer to is: “Et tunc ad argumentum: ‘tu dubitas propositionem falsam, igitur dubitas propositionem falsam vel aliquid esse non aliquid’, nego consequentiam. Et cum dicitur: ‘...a parte disiuncti ad totum disiunctum’, dico quod non valet consequentia quando modus praecedens disiunctum determinat sicut est in proposito,” PAUL OF VENICE 1981, 92.

90 “Ita in proposito dicatur quod sive illae convertantur sive non convertantur materialiter vel formaliter : ‘Hoc est Sortes’ et ‘Hoc est Sortes vel Plato’, significatum primum unius non est significatum primum alterius, quia significatum primum primae est significatum simplex ortum a simplicibus extremis, et significatum primum secundae est significatum disiunctum ortum a terminis compositis, ut patet. Non tamen nego quin si-

Pardo's solution to the sophism has two notable features. First, similar to his predecessors, he takes it as an opportunity to address sentential semantics. As opposed to Heytesbury and Paul of Venice, who are primarily concerned with the formal-semantic aspects of the problem, Pardo mentions the ontological aspect of the problem. He introduces two alternative views of sentential meaning or '*complexe significabile*', which were discussed in preceding parts of *Medulla dyalectices*.⁹¹ However, the distinction plays no significant logical role in the solution to the sophism: Pardo translates the problem into the terminist framework by introducing appellation to explain the restrictions blocking the opening argument and demonstrates its function by distinguishing between *de re* and *de dicto* contexts generated by the verb 'signify'.⁹² It seems possible to label Pardo's approach as a Continental solution to a British problem. Originally, the problem consisted in the consequences of Bradwar-

nificatum unius significetur a reliqua et econtra, sed non primarie," PAUL OF VENICE 1981, 98.

91 For an overview of the debate on *complexe significabilia* in the post-medieval period, see NUCHELMANS 1980, 45–73 and (among her other works), ASHWORTH 1978, 81–121, ASHWORTH 1981, 61–96, and a recent analysis of Pardo's position is presented in PÉREZ-ILZARBE 2016, 512–531 (who defended a doctoral dissertation on Pardo's sentential semantics).

92 "Ad hoc argumentum brevis patet responsio. Nam si ponantur complexe significabilia, ut ponit primus modus dicendi, illa est falsa: '*illa propositio "hoc est Socrates" significat hoc esse Socratem vel Platonem*', neque valet consequentia: '*significat hoc esse Socratem, ergo significat hoc esse Socratem vel Platonem*'. Quando enim arguitur a parte disiuncti ad totum disiunctum, servandum est ut reliqui termini respectu quorum non arguitur a parte disiuncti ad totum disiunctum pro eodem teneantur in antecedente et in consequente, quod non est in proposito. Nam quando dico: '*hec propositio "hoc est Socrates" significat hoc esse Socratem*', *ly significat* appellat significationem istius '*complexe significabile hoc esse Socratem*', sed cum dicitur in consequente '*significat hoc esse Socratem vel Platonem*', *ly significat* appellat significationem illius '*complexe significabile hoc esse Socratem vel Platonem*', ideo *ly significat* non eodem modo stat in antecedente et in consequente. Si etiam teneatur secundus modus de complexe significabilibus, tenendo restrictionem illam rigorosam dictam negaretur etiam illam consequentiam esse bonam propter variationem appellationis de *ly significat*. Ex isto antecedente potest inferri istud consequens: '*ergo hoc esse Socratem vel Platonem significat*'. Ultra tamen illa potest dici quod illa propositio: '*hec propositio significat Socratem vel Platonem*' potest distingui: aut est disiunctiva, et sic est vera, aut de disiuncto extremo, et sic est falsa, tenendo restrictionem supradictam," JEROME PARDO 1505, fol. 101rb.

dinian semantics for Heytesburian epistemic logic. The problem would later become further complicated by introducing propositional ontology, but that did not, ultimately, influence the general strategy. As such, Pardo's approach exemplifies a more general trend which Ashworth, in the context of the analysis of intensional and hyperintensional verbs (such as 'promise' and 'require'), described as preferring Buridan to Heytesbury.⁹³

5. Conclusion

The analysis of epistemic sophisms discussed in John Mair's circle contributes to the exploration of sixteenth-century Parisian logic. The general trend can, in this particular case, be summarised as 'British logic mediated by Italian commentators'. While the original source for most of the problems discussed above was William Heytesbury, the actual sources for John Mair's circle appear to be Paul of Venice and Cajetan of Thiene.

There are two notable terminological peculiarities pertaining to the Parisian authors as compared with their British and Italian sources. First, there is Pardo's use of *appellatio* in his solution to the problems associated with Bradwardinian semantics. Second, there is a common tendency to replace belief or knowledge with assent.⁹⁴ Both tendencies appear to be instances of bringing Continental tools into British debates. Furthermore, there is an interesting trend for naturalising logic and epistemology, such that the solution of some of the sophisms becomes an empirical (and, in some cases,

⁹³ See ASHWORTH 1974(1), 148 and ASHWORTH 1976, 71-72.

⁹⁴ As an example, this is Heytesbury's working definition of knowledge: "...scire non est aliud quam sine hesitatione apprehendere veritatem, idest credere sine hesitatione quod ita sit et cum hoc quod ita sit ex parte rei..." WILLIAM HEYTESBURY 1494, fol. 13vb. In contrast, Pardo used the following formulation of the same problem: "Uno modo accipitur scientia pro assensu propositionis vere vel melius illius significati sine formidine de opposito..." JEROME PARDO 1505, fol. 96ra. For the relations of such formulations to the fourteenth-century Parisian tradition, see LAGERLUND 2019, *passim*.

quantifiable) problem.

This paper has some loose ends. The analysis of the three bodies scenario showed a common distortion of the original argument and what appears to be a mis-reference to Cajetan of Thiene. This is suggestive of either missing sources or covert connections. The first problem could be solved by broadening the corpus, assuming that such a source or group of sources was a written text rather than, say, an undocumented lecture attended by one of the authors, and was not lost. The second problem could only be solved conjecturally based on biographical and institutional data. As a hypothesis, such a mis-reference could have originated between Mair and Coronel and was first published by Coronel; one of these two could also have been responsible for its further dissemination.

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