Paul of Venice's Theory of Quantification and Measurement of Properties

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Abstract: This paper analyzes Paul of Venice's theory of measurement of natural properties and changes. The main sections of the paper correspond to Paul's analysis of the three types of accidental changes (local motion: section II; augmentation: section III; alteration and qualities: section IV), for which the Augustinian philosopher sought to provide rules of measurement. It appears that Paul achieved an original synthesis borrowing from both Parisian (Albert of Saxony in particular) and Oxfordian sources (especially Richard Swineshead). It is also argued that, on top of this theoretical synthesis, Paul managed to elaborate a quite original theory of intensive properties that marks him out not only from the nominalist framework of his Parisian sources but also from the usual realist treatments of the problem. Finally, it is shown that, to a certain extent, Paul undertook to apply the mathematical and logical tools inherited from the *Calculatores* tradition to empirical problems of natural philosophy, leading to reevaluate his role in the evolution of scientific thought in early 15th-century Italy (section V).

Keywords: Paul of Venice; Oxford Calculators; motion; speed; intension of forms.

1. Introduction

Paul of Venice (ca. 1369–1429) is acknowledged as one the most important philosophers of the late Middle Ages. In the Renaissance, he enjoyed a solid reputation, evidenced by the numerous editions of his writings. Recent studies on Paul of Venice have confirmed the value of his thought and established beyond dispute the depth of his views in metaphysics and logic.[1] The many

* The research for this article was funded by the Swedish Research Council (grant agreement no. 2019-02777).

1 The secondary literature on Paul of Venice's metaphysics and logic is extensive; see in priority Conti 1996; Conti 1982a; Conti 1982b; Galluzzo 2013, 385–466; Amerini 2004.

Noctua, anno IX, n. 2, 2022, ISSN 2284-1180
DOI: 10.14640/NoctuaIX5

philosophical influences that shaped Paul's thought are now better understood. Yet, focusing mostly on his metaphysics and his logic, scholars have often left behind Paul's physical theories.[2] This situation may partly stem from derogatory judgments coming from early historians of science. For instance, P. Duhem considered Paul as a poor thinker when it comes to natural philosophy, depicting him as a mere "plagiarist" of Restoro d'Arezzo,[3] while L. Thorndike, with more moderation, underlined the rather traditional content of his cosmological theories.[4] Such judgments, however, depended on a certain conception of scientific progress. Undeniably, Paul of Venice did not increase scientific knowledge by providing new explanations for geological phenomena or astronomical facts, for example. But as this paper aims to show, his contributions to the evolution of natural philosophy lie elsewhere. It is nowadays known that late medieval physics benefitted from many conceptual innovations from the 14th century onward.[5] A growing interest in the quantification of natural phenomena, i.e. in the use of mathematical concepts to describe physical processes, led a growing number of thinkers to investigate from a new point of view motion and other natural processes. In particular, it is now better appreciated how the 'Oxford Calculators' – the most influential school of this trend (comprising Thomas Bradwardine, Richard Swineshead, William Heytesbury and John Dumbleton, among others) – produced scientific results leading to a gradual distancing from the traditional way of studying nature, which were transmitted and taken up a few centuries later by Galileo and other scientists.[6]

2 Among the few recent studies on Paul's natural philosophy, see Bottin 1984; Majcherek 2020.

3 Duhem 1913–1959, vol. 4, 209–210.

4 Thorndike 1929, 195–232.

5 Maier 1949–1958, vol. 2, 1–109; vol. 1, 81–131; vol. 3, 255–384; Sylla 1991; Sylla 1973; Sylla 1971. See more recently Jung, Podkoński 2020.

6 Di Liscia, Sylla; Sylla 1997. More specifically for the Italian context, see Lewis 1980; Podkoński 2013.

Paul of Venice was an important figure in the evolution of this new way of doing physics. As will appear, he played an important role in the transmission of new physical methods and, more precisely, new conceptual techniques for quantifying and measuring physical phenomena. His writings reveal a complex theoretical system of measurement synthesized from philosophical materials inherited from the two most important universities of the 14th century, namely Paris and Oxford. Although S. Caroti highlighted some aspects of his views,[7] Paul's complete theory of the quantification and measurement of natural properties has never been described in detail. The present paper aims to provide an overview of this theory, to highlight his influences regarding the different aspects of his positions and, thus, to make possible a better understanding of his role in the diffusion of new ideas in physics in the Italian context. The following sections provide a study of Paul's method for quantifying natural properties according to the three types of motions he acknowledged, as a natural philosopher still largely indebted to an Aristotelian framework.

The next part (section II) of this study exposes how Paul uses conceptual tools inherited from the Oxford Calculators for the analysis of local motion and speed. After a brief analysis of his views on quantitative change (section III), it will be shown that, for the case of alteration, Paul designed an original theory of the nature of qualities, which served as a basis for his method of measuring single qualities and mixed bodies (section IV). Different applications of these tools to empirical problems of natural philosophy make it possible to establish Paul's pivotal role in the transmission and development of the new 14th-century physics in late medieval Italy (section V).

7 See the remarks on Paul's theory of the quantification of qualities in Caroti 2012; for understanding the discussion of the problem in the Italian context, see Caroti 2014.

2. Theory of motion and measurement of speed

2.1 Rules of motion according to its cause (tanquam penes causam)

Most of the 14th-century discussions over the rules of motion were stimulated by Thomas Bradwardine's *Tractatus de proportionibus*, which was published in 1328 and still exerted an important influence on 15th-century natural philosophers. In this work, the English theologian formulated what is commonly referred to as 'Bradwardine's law' by historians of science, although this law may have been first formulated by earlier authors.[8] This rule, accepted by Paul of Venice, describes the ratios between the speeds or velocities (*velocitates*) of two bodies from the point of view of their cause (*tanquam penes causam*), i.e. considering the force (or moving power) and the resistance (or resistive power) involved as causes of motion. In his treatise on the proportions of motion, Bradwardine argued that the ratio of velocities between two moving bodies is proportional according to a geometrical proportion to the ratio between forces and resistances of the two bodies. Using symbolic notation, if S denotes speed, F force and R resistance, Bradwardine's law states that:

$$S1 : S2 = (F1 : R1) : (F2 : R2)$$

This law differs from Aristotle's way of characterizing the relation between speed, force and resistance. In Book 7 of his *Physics*, Aristotle suggested that the speed of a body is doubled when the force exerted upon it is doubled. On this basis, Aristotle seems to admit that, more generally, the speed of a body is simply proportional (i.e. according to an arithmetical proportion) to the force exerted upon it when the resistance remains constant.[9] As such (al-

8 The paternity of this law is still discussed by historians, and has been variously attributed to Arnald of Villanova or Richard Kilvington. See McVaugh 1967; Drake 1973; Jung, Podkoński 2008.

9 Aristotle, *Physics*, VII, 5, 250a4–6, 250a25–28.

though Aristotle does not explicitly express it in this general form) the Aristotelian analysis of motion according to its causes was often interpreted as the relation S = F/R. Bradwardine's use of a geometrical proportion rather than an arithmetical one was intended to avoid the physical paradoxes entailed by the 'Aristotelian rule.' For instance, some consequences of the Aristotelian rule, some of which were anticipated by Aristotle himself, are that a man would be able to move a heavy object like a boat that twenty men can move, although more slowly, and that more generally a body of an infinite resistance would be moved in an infinitely long time by any given force – facts obviously denied by experience.

Bradwardine's law avoids these troubles. In an anachronistic fashion (due to the lack of a proper concept of mathematical function in the Middle Ages), we would write it as:

$$Sn = \left(\frac{F}{R}\right)^n$$

This law implies that the speed a body will be halved only when the ratio between its moving force and its resistance will be reduced to its square root and that, more generally, the moving force exerted upon a body moves it with a speed equal to S/n only when the resistance is:

$$\sqrt[n]{R}$$

Thus, a force can only move a body whose resistance is strictly inferior to it. Acknowledging the superiority of Bradwardine's law, Paul underlines that the speed of a body moved with a force of degree 4 and a resistance of degree 1 will be doubled not if the force acquires the degree 8, but only if it reaches the degree 16, so that the ratio 4 : 1 is squared. Under this assumption, the 'Aristotelian rule' according to which the speed of a body is doubled when the force exerted upon it is doubled becomes false except for the *only* case where the ratio between force and resistance is 2 : 1.[10] Generally, the proportion between the velocities of two bodies does not follow the proportion between the forces exerted upon them, nor the proportion between their resistances.[11] To establish this point, Paul draws on Euclid's theory of proportion and, more precisely, on the rules for compounding proportions.[12] Paul recalls that, if three integers A, B and C are such that $A > B > C$, then $A : C = A : B \bullet B : C$, where $\bullet$ denotes the operation of compounding two proportions. Given this law, a ratio A : C is equal either to $(A : B)^{2/1}$ or to $(B : C)^{2/1}$ only when $A : B = B : C$. Thus, the only case where doubling the motive power entails doubling the speed is when the ratio between motive powers is 2 : 1.

Whereas most of Bradwardine's contemporaries and the main representatives of the Parisian and Oxfordian schools followed him, several thinkers from the 14th century onward – for instance Blasius of Parma, one notable exception in the Italian tradition – criticized the implications of his rule.[13] Paul has a more conservative and accommodating attitude on this point. Not only does he entirely accept Bradwardine's rule, but his respectful attitude toward Averroes even leads him to attribute this view to the Commentator himself (conceding, however, that one has to interpret it as the true meaning of the relevant passage of Averroes' commentary on the *Physics*).[14] This fact should not surprise us: Bradwardine himself, when formulating his

10 PAUL OF VENICE 1499, VII, tex. com. 36: "Sequitur quod si motoris ad mobile est proportio maior quam dupla ipsum non movebit medietatem mobilis in duplo precise, sed minus quam in duplo." Cf. PAUL OF VENICE 1521, Physic., c. 33, 26vb–27rb.
11 PAUL OF VENICE 1499, VII, tex. com. 36. Cf. PAUL OF VENICE 1521, Physic., c. 32, 25vb–26ra.
12 PAUL OF VENICE 1499, VII, tex. com. 36. Cf. PAUL OF VENICE 1521, Physic., c. 33, 26rb–27va.
13 On this point, see ROMMEVAUX-TANI 2008.
14 PAUL OF VENICE 1499, VII, tex. com. 35.

rule of motion, quoted Averroes as a supporting authority.[15] Paul's adhesion to Bradwardine's law, at any rate, remains typical of the most common opinion of his time regarding the rules of motion *tanquam penes causam*. Even though independent thinkers like Giovanni Marliani will reject this law shortly after him, many influential Italian thinkers long after Paul's time, like Benedetto Vittori or (although only to a limited extent) Alessandro Achillini, were still regarding this law as true.[16]

2.2 Measurement of motion according to its effect (**tanquam penes effectum***)*

The discussion of the rules of velocities according to its cause were not the only way of analyzing motion from a quantitative point of view. Besides this approach, a growing interest for the analysis of motion according to its effect (*tanquam penes effectum*) became a central theme from the 14th century onward. Analyzing motion according to its effect meant, in this context, determining the relative velocities of different bodies not on the basis of the forces exerted upon them, but comparing the spaces travelled during a certain time. Remarkably, whereas Bradwardine's law was commonly accepted by 14th-century natural philosophers, the rules for measuring motion according to its effect were a greater source of disagreement.

Indeed, different parts of a moving body could be taken as a reference for measuring its velocity: given an extended body, for instance, should we consider its surface as the reference for measuring its velocity or one of its points? And in the latter hypothesis, which point should we take as a reference? These questions formed a hotly debated topic since the choice of one convention over others could lead to problematic results, usually

15 THOMAS BRADWARDINE 1955, 110.
16 ALESSANDRO ACHILLINI 1545, 192rb–va. See SYLLA 2008; BIARD 2008.

presented in the *Calculatores* tradition as series of paradoxical conclusions involving infinite speeds, instantaneous motions or contradictory ones. The topic was all the more intricate that the question could be asked for rectilinear motions as well as curvilinear ones, and that different conventions could be used for both types of motion, as evidenced by Paul's own position on this problem.

Regarding rectilinear motion, Paul claims that the speed of a body moving in such a way should be measured by its middle point, and not according to its fastest point. By stating this rule, Paul rejects Bradwardine's opinion according to which the speed of a moving body (in the case of local motion) must be measured in all cases by its fastest point.[17] According to Paul's convention, if Socrates and Plato were competing at a race and ran at the same speed, the fact that Socrates would stretch his arm at the finish line would not be sufficient to say that he ran faster.[18]

Paul nonetheless subscribes to the convention of taking the fastest point for measuring speed in the case of circular motion, which corresponds to the solution Thomas Bradwardine and William Heytesbury had chosen on this problem. Paul's main source here, however, is Albert of Saxony, who endorsed the same position in his influential *Treatise on Proportions* (itself largely indebted to Bradwardine's own treatise).[19] Paul explicitly quotes Albert and takes up the complex position defended by the Parisian Master. Indeed, Albert refused the use of a single principle for measuring the speed of bodies

17 THOMAS BRADWARDINE 1955, 130, l. 128–129: "Cuiuslibet motus localis, velocitas secundum maximum spatium lineale ab aliquo puncto sui moti descriptum accipitur."
18 PAUL OF VENICE 1499, VI, tex. com. 14: "Si Sortes et Plato currerent equevelociter et circa finem spatii Sortes extenderet brachium, non propter hoc diceretur tunc moveri velocius Sortes quam Plato, licet aliquis punctus velocissime motus in Sorte velocius moveretur quam punctus velocissime motus in Platone." Cf. PAUL OF VENICE 1521, Physic., c. 34, 27va.
19 ALBERT OF SAXONY 1971, 70, l. 515–519.

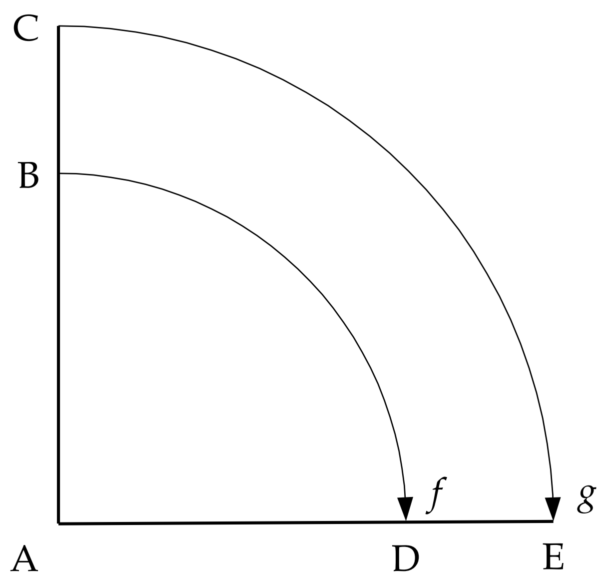
according to their effect. For rectilinear motion, Albert also accepted that the middle point of a moving body should be used as the reference for measuring its velocity.[20] But for circular motion, Albert concluded that the fastest point should be used as the reference point on the basis that the speed of a body must be predicable of any part of it. Thus, the maximum speed attained by a body, in the case of rotation, must correspond to the point farthest from the center of the radius, i.e. to the point on the circumference of the circle (or circular trajectory). Paul accepts Albert's reasoning, leading him also to reject Gerard of Brussel's theorem according to which the average speed of a rotating radius corresponds to the speed of its mean point.[21] Paul acknowledges that the latter convention is similar to the 'mean speed theorem' according to which the total speed of a uniformly difform motion (i.e. uniformly accelerated motion) is equivalent to half the final speed of the accelerated body. The reason why a similar convention does not apply in the case of rotation is that the mean speed theorem is only valid according to Paul with respect to time, not with respect to the subject and its extension, so that the speed of a rotating body cannot be measured by its mean point.

Paul also takes up from Albert of Saxony the distinction between circular motion (*motus circularis*), which as we have seen must be measured by the fastest point of the mobile, and the motion of rotation as such – *motus circuitionis* – which we could call angular speed in anachronistic terms. The *motus circuitionis* must be measured by the angle formed by the radius and its origin in a given period of time. Given these two types of speed related to rotation, it is possible to say that two things are moved (*moveri*) with different velocities but rotate (*circuire*) at the same velocity, like a point of the equinox

20 ALBERT OF SAXONY 1971, 68, l. 459–461.
21 PAUL OF VENICE 1499, VI, tex. com. 14; PAUL OF VENICE 1521, Physic., c. 35, 28rb, 3a concl.

and a point of the Arctic Circle (given that their circles have the same center).[22] As Paul puts it, these two motions are really identical but they are distinct aspects (*sunt diversarum rationum*) of the same rotation and, as such, the *motus circuitionis* can be compared neither to rectilinear motion nor to circular motion, for an angle cannot be compared to a line.[23]

Taking the figure below as an illustration, the speed of circular motion (*motus circularis*) of a body *f* starting from B and reaching D after a given time must be measured by the linear distance BD, whereas the speed of its rotating motion must be measured by the degree $n°$ of the angle $\widehat{BAD}$. If another a body *g* starts from C and reaches E at the same time, it will have the same rotating speed (*motus circuitionis*) measured by the angle but $\widehat{CAE}$ will move faster according to the speed of its circular motion (the latter being measured by the linear distance CE).



As the summary table below shows, Paul's view on the measure of motion

22 PAUL OF VENICE 1499, VI, tex. com. 14: "Ideo possible est duo inequevelociter moveri et equevelociter circuire."
23 PAUL OF VENICE 1499, VI, tex. com. 14. Cf. ALBERT OF SAXONY 1971, 70, l. 549–551.

tanquam penes effectum is strictly identical with Albert of Saxony's doctrine which, to a certain extent, represents a refinement of Bradwardine's views.

	Gerard of Brussels	**Thomas Bradwardine**	**Albert of Saxony/Paul of Venice**
Rectilinear motion	ø	Fastest point	Mean point
Circular motion (*motus circularis*)	Mean point	Fastest point	Fastest point
Rotating motion (*motus circuitionis*)	ø	ø	Angular degree

3. Speed of quantitative change

Compared to local motion and alteration, rules for quantitative changes receive a rather brief analysis in Paul's works. Paul chooses to measure augmentation properly said by taking as the reference the absolute acquired quantity in a given time.[24] As a consequence, he rejects a type of position that had been defended by William Heytesbury and later criticized by Richard Swineshead in the *Calculationes*, namely that the speed of augmentation is to be measured by a *proportion* between quantities. This type of position comes in two versions. The first one, whose champion is not mentioned by name, chooses to measure the speed of augmentation by the proportion between the acquired quantity and the preexisting one. This position leads to the problematic situation that a body whose quantity is precisely doubled (so in which the acquired quantity M is equal to the preexisting one N) will not be comparable to any other augmentation. The idea that equal proportions cannot be

24 Paul of Venice 1521, Physic., c. 36, 29ra, 4a concl.

compared through any ratio to inequal proportions, stemming from the very definition of proportionality, was widely accepted due to Bradwardine's influential *Treatise on Proportions*, including by Albert of Saxony. There exists a proportion between two things M and N, given that M < B, if and only if there is some *p* such that M*p* > B. Since this condition cannot be met for the proportion of equality (M = N), this proportion cannot be compared to any proportion of greater or lesser inequality.[25]

Paul also rejects the view that the speed of augmentation is to be measured by the ratio (M + N)/N, which leads to the paradoxical conclusion that something augmenting from the non-degree of quantity to any finite quantity will increase with an infinite speed, since in this case the ratio will be infinite.[26] This position corresponds to the view endorsed by Albert of Saxony in his *Treatise on Proportions*, and to the first position (i.e. Heytesbury's) criticized by Richard Swineshead in the sixth treatise of the *Calculationes*.[27]

In the face of these difficulties, Paul measures the speed of augmentation by the acquired quantity, namely M. Due to the very brief character of the section devoted to this point, it remains difficult to compare it to Richard Swineshead's solution. Richard was inclined toward a similar position, which had already been defended by Roger Swineshead, but Richard took care to precise that only the *total* quantity acquired at the end of a quantitative change should be taken as a reference. This proviso added to Roger Swineshead's view meant that if the subject loses some part A while acquiring M, this fact should also be taken into account, leading to a relation such as: (M +N – A) – N, i.e. M – A.[28] In any case, it must be noted that the formu-

25 THOMAS BRADWARDINE 1955, 80: "Proportione aequalitatis nulla proportio est maior vel minor."
26 PAUL OF VENICE 1521, Physic., c. 36, 28vb, 1a concl.
27 ALBERT OF SAXONY 1971, 71, l. 568–590; RICHARD SWINESHEAD 1520, VI, 22ra–24va.
28 RICHARD SWINESHEAD 1520, VI, 24vb.

lation of Paul's solution, i.e. that speed in this case depends on the "absolute acquired quantity" was expressly rejected by Albert.[29] The only difference between Richard Swineshead and Paul on this point lies in the fact that, unlike the Calculator, Paul does not care to discuss the more complex cases where a subject undergoes different quantitative changes at the same time. Despite this fact, he also refuses proportionality as a measurement method for this type of motion, for such a mathematical relation leads according to him to various paradoxes, a fact bringing support to the much simpler use of a mere difference relation. Taking again N as the initial quantity, M as the acquired one and A as a simultaneously lost quantity (as taken into account by Richard Swineshead), the difference between Paul's position and the alternative options he discusses may be summarized as follows:

?	**Richard Swineshead**	**Albert of Saxony (and William Heytesbury)**	**Paul of Venice (and Roger Swineshead)**
M/N	(M + N – A) – A	(M + N)/N	M

4. Quantification of qualities

As a natural philosopher whose conceptual framework is still profoundly Aristotelian, Paul considers alteration as a central type of natural change. He grants special attention to the topic of intensive properties or, in medieval terms, to the problem of *intensio et remissio formarum*, an important chapter of the new physics inherited from the 14th century. A large part of Paul's reflections on natural motion is devoted to the problem of understanding how intensive variations can occur, and how to *measure* them. It is plainly obvious that a glass of water can become more or less hot, or a leaf more or less green.

29 ALBERT OF SAXONY 1971, 71, l. 557–558.

Explaining these common observations was the subject of intense debates in the late Middle Ages. Indeed, medieval natural philosophers following the Aristotelian account of change tended to explain change by the replacement of one form (i.e. one property like hotness) by another form (like coldness) in the same substrate. They also generally rejected corpuscular and atomist conceptions of matter explaining sensible properties and their modifications by interactions between particles. How then is it possible to explain the variation of one and the same property? Is it possible, and if so how, to measure them? Before getting to Paul's view on this last problem, an exposition of his ontological theory of intensive properties will help us understand his theoretical assumptions.

4.1 Ontology of intensive properties

4.1.1 Theory of intensive variations

In the late Middle Ages, the famous debate over the explanation of intensive properties was not only a problem of natural philosophy. Originating in theological context with the 17th distinction of Peter Lombard's first book of the *Sentences*, which stated that charity can increase in the soul of a human being, the discussion over the intensity of forms was soon extended in the 14th century to all types of qualities, including cognitive habits, moral and theological virtues, as well as many types of physical properties.[30] Four main theories, or rather families of theories, were discussed in the debates over the cause of the intensive variations of qualities:

1. The theories of participation

30 Paul himself employs the terms proper to intensive variations (like "degrees" or "latitude") for abstract objects, for instance *per se notum* or *per se verum*. Cf. PAUL OF VENICE 1499, II, tex. com. 6.

2. The addition theories
3. The admixture theories
4. The succession theories

The differences between these various theories are now well documented. Suffice it here to briefly summarize these different conceptions and underline their main advantages and drawbacks according to Paul.[31] The theories of participation (a) were mainly inspired by Thomas Aquinas and Giles of Rome, notwithstanding important divergences between these two authors. According to these theories, a subject acquires a more intense quality when it participates more fully in this quality. The Platonist tone of the term 'participation' as well as the rather abstract conception of what a quality is underlying this solution explain that it was often rejected by 14th-century philosophers, inclined toward more concrete – if not empiricist – explanations of intensive variations. Paul does not take care to refute this view, despite his overall respectful attitude toward Giles of Rome, the most prominent figure of the Augustinian order in the 13th century, whom he extensively quotes in his writings, including his exposition on the *Physics*. Given Paul's metaphysical standpoint, quite different from Aquinas' and Giles', such a theory of participation could not be considered as a serious candidate for explaining intensive variations.

A similar remark applies to the succession theory (d), famously defended at the beginning of the 14th century by Walter Burley who, along with Averroes, stands as the most quoted author in Paul's exposition on Aristotle's *Physics*. Not just an important source for Paul's natural philosophy, Burley is foremost a crucial inspiration for his views on metaphysics and especially on

31 For comparative analyses of these theories, see MAIER 1949–1958, vol. 2, 1–109; SYLLA 1991; SYLLA 1973; JUNG 2011; ROUDAUT 2021.

universals. Burley advocated a theory of intensive properties according to which what is perceived as the intensification or remission of a quality is not actually a process affecting one and the same numerical quality, but a series of gradual replacements of one form by another form at each instant of the alteration (each form being *indivisible* and *instantaneous*). In a way, Burley conceptualized intensive alteration on the model of local motion: just like a moving body is located at a different place at every instant of its motion, a subject acquires another quality at every instant of its alteration. Without refuting it directly, Paul rejects this view by assuming that a quality can be divided into different intensive parts – a point incompatible with Burley's view.[32] What is more, an intensible quality is infinitely divisible according to Paul. Such a quality has no natural minimum, neither according to extension nor according to intension.[33]

The admixture theory (c) tried to explain the intensive variation of a quality by the presence of its contrary quality in the same subject. A body is more intensely hot than another when it is less mixed with coldness, just like a whiter surface has less blackness than a grey one. In this crude version, the admixture theory was largely rejected by scholastic authors – including Paul – on the basis that it contradicts the principle according to which contraries cannot co-exist within the same subject.[34] However, from the 14th century onward, a certain aspect of it was integrated into what was arguably the most popular explanation of intensive phenomena: the addition theory (b). The addition theory accounted for intensive properties on the model of quantitative

32 Paul of Venice 1521, Metaph., c. 17, 132vb: "Sicut quantitas habet partes quantitativas, qualitas habet partes intensivas […], qualitas intensibilis et remissibilis est divisibilis in duas medietates, in tres tertias, in quatuor quartas, et sic in infinitum in partes equales intensivas."

33 Paul of Venice 1521, Physic., c. 1, 3rb.

34 Paul of Venice 2000, d. 17, q. 4, a. 2, 395.

change. Just like quantities increase by an additive process, a quality increases intensively when it acquires more degrees. From the point of view of the addition theory, there is an addition or a loss of intensive degrees whenever a quality is intensified or remitted, respectively. Toward the mid-14th century, a hybrid version of the two last theories was finely articulated by influential philosophers like John Buridan or Marsilius of Inghen.[35] According to this version of the addition theory, which Paul endorses, the intensity of a quality results from the sum of contrary degrees (but not contrary *forms* or *qualities*). For instance, given a scale of temperature in which the highest degree of hotness is 8 and the minimum is 0, a temperate body will contain 4 degrees of hotness and 4 degrees of coldness. A warmer body having 7 degrees of hotness will only contain 1 degree of coldness, so that the total sum of degrees always remains constant (8 degrees).[36] This (new) version of the addition theory is not equivalent to the 'crude' version of the admixture theory, because two completely actualized qualities cannot exist in the same subject.[37] For instance, a body hot at the degree 8 must have 0 degree of coldness (given that the sum must remain equal to 8 degrees in total). Paul supports this view by ontological distinctions. Whereas *privative* contraries like light and shadow or motion and rest cannot co-exist in the same subject (from which it can be deduced that privations are indivisible and have no latitude), *positive* contraries like whiteness and blackness can do so. But two cases must

35 See CAROTI 2004; CLAGETT 1941, 37–38.

36 PAUL OF VENICE 1499, V, tex. com. 52. "Pro solutione omnium istorum est primo notandum secundum opinantes contraria esse simul quod semper complet numerus graduum contrariarum in eodem subiecto, ita quod signata omni latitudine qualitatum per numerum, ut 8, si non est in subiecto caliditas ut 8 sed remissa, ipsa completur frigiditate tanta ex qua et caliditate remissa fit unum aggregatum ut 8. Verbi gratia […] cum caliditate ut 7 stat unus gradus frigiditatis et non plures."

37 Paul is well aware of the need to refine the notion of contrariety. In his exposition on the *Physics*, he distinguishes six degrees or types of contrariety; cf. PAUL OF VENICE 1499, V, tex. com. 55. See also PAUL OF VENICE 1521, Physic., c. 28–30, 23ra–25ra.

be distinguished. When two positive contraries are not the extreme species of a genus, the sum of their degrees can exceed the number of degrees attributed as a convention for measuring some latitude (like 8 degrees). For instance, a very pale body has both paleness and whiteness, and since paleness itself contains a certain part of whiteness, the sum of their degrees exceeds 8. But the sum of degrees of two extreme contraries like whiteness and blackness must always be equal to 8 degrees.[38] This claim is strictly equivalent to the positions held by the main representatives of the Parisian school of the 14th century.[39]

This way of conceiving the compresence of contrary qualities accounts for physical processes otherwise unexplainable. According to Paul, the fact that warm water has a cooling effect on hotter water can only be explained if one admits the presence of degrees of coldness in the former, whereas a light of a certain degree of intensity is not remitted due to the presence of a less intense light.[40] Thus, Paul recognizes a certain type of admixture in intensive phenomena, even if he also retains from the addition theory the idea that intensive degrees possess additive properties analogous to quantities.[41] Intensity is indeed similar to quantity, even if unlike extension, which belongs essentially to continuous quantity and, hence, cannot be separated from it, intension is extrinsic to quality and only exists when a quality informs a subject. Thus, unlike continuous quantity for which one can differentiate indeterminate and determinate extension, there is only *determinate* intensity.[42] But

38 PAUL OF VENICE 1521, Physic., c. 30, 24va.
39 See for instance MARSILIUS OF INGHEN 1521, III, 10ra: "Omnia corpora naturalia habent eque multos gradus qualitatum primarum. Patet, quia si haberent pauciores caliditatis, haberent plures frigiditatis, et econtra. Similiter in humiditate et siccitate, quia quando unus gradus unius remittitur tunc unus alterius introducitur."
40 PAUL OF VENICE 1521, Physic., c. 30, 24vb.
41 PAUL OF VENICE 1521, De gen. et corr., c. 1, 48rb, 4a concl.
42 PAUL OF VENICE 1521, Metaph., c. 18, 134rb.

it must also be noted that according to Paul, strictly speaking, the *subject* is what is intensified, and not the quality itself, since when a quality acquires a new degree, it is not the same anymore.[43]

Interestingly, Paul seems willing to provide an *a priori* deduction of the number of the elements from this conception of intensive qualities. Since the sum of contraries cannot exceed the total degrees of a latitude, they cannot be both above the middle degree (so degree 4), whereas an element requires two qualities above the middle degree. Thus, there are four possible combinations between the four elemental qualities, and two impossible (hotness/coldness; dryness/wetness), which explains that there cannot be more than four elements.[44]

This additive model must be restricted according to Paul to accidental forms and, as we will see, cannot be extended to the case of substantial ones. Subscribing to a version of the plurality of substantial forms thesis, Paul feels the need to justify this point and explains that the addition of several substantial forms of the same reason cannot adequately take place in one and the same subject. For this reason, the soul of a human being cannot be said to be more 'soul' than another one, whereas one whiteness can be said to be whiter than another because several accidental forms can adequately be in the same subject.[45] This remark, despite creating some confusion as to the categorial nature of *what* is added in intensive variations (a form? or a degree?), underlies the fundamentally additive character of these physical phenomena in Paul's view.

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43 PAUL OF VENICE 1521, De gen. et corr., c. 1, 48rb, 2a concl.

44 PAUL OF VENICE 1521, De gen. et corr., c. 8, 52va–vb, 4a concl. Cf. JOHN BURIDAN 2010, II, q. 3, 202–203.

45 PAUL OF VENICE 1499, VI, tex. com. 45. See also PAUL OF VENICE 2000, d. 17, q. 4, a. 3, ad 1m, 402.

4.1.2 A realist ontology of degrees

Despite sharing a theory designed by influential masters of the Parisian school, Paul has an ontological conception of intensities quite different from his sources. Indeed, in the nominalist approach proper to the Parisian school, there is no real distinction between a quality and its degree of intensity. Buridan, one of the most radical nominalist representatives of the Parisian school, stresses in his *Questions on the Physics* that a degree is of the 'same nature' (*eiusdem rationis*) as the quality of which it is a degree, being not really distinct from it. A degree of heat is heat.[46] Buridan denies that any real distinction is entailed by the two phrases 'white' (*album*) and 'being white' (*esse album*). The difference between those terms is only semantical: whereas *album* only refers to the body informed by an individual quality (*albedo*), the phrase '*esse album*' refers to the same thing but also connotes the quality informing it.[47] In this nominalist interpretation of terms referring to intensive properties, the 'degree' is a term denoting a particular quality (hotness) which connotes the quantity of parts present in this quality (the degrees). The terms 'degrees', 'intensity', and 'quality' are different nominal descriptions of the same thing and in no way refer to really distinct properties.[48]

By contrast, Paul believes that the intensity of a quality is a property really distinct from it. As he puts it, a quality can be intensified and remitted, but the intensity itself cannot. Thus, the predicate 'intensifiable' can be attributed to a quality, but not to its intensity, which shows according to Paul that they cannot be identical:

46 John Buridan 2015, q. 4, 42, l. 5–6; 43, l. 10–11. See the similar remarks of (Ps.-?) Marsilius of Inghen 1518, III, q. 3, 37rb.
47 John Buridan 1518, IV, q. 6, 16va–17vb, esp. 17rb.
48 See Biard 2002.

> Third conclusion. Every absolute intension is really distinct from its quality. […] Fourth conclusion. An absolute intension is a passion of an intensible and remissible quality. This conclusion is obvious, just like absolute dimensions are passions of continuous quantities. This passion however is not essential, since the quality can be multiplied while the intensity remains constant and conversely.[49]

Paul's realist conception of intensities is confirmed by the fact that an intensive degree must be defined according to him as an absolute accident, not a relative one (*respectivum*).[50] Thus, the intensity predicable of one quality does not essentially depend on a comparison between different qualities, according to which this particular quality could be said more or less intense.

Key to Paul's argument here is that real identity implies the possible substitution of essential predicates. As well established by A. Conti's works quoted at the beginning of this study, Paul's general metaphysics is governed by realist convictions, inherited not only from Duns Scotus and Burley but also from the Oxfordian realist school of the second half of the 14th century, initiated by John Wyclif and continued by his followers (John Sharpe, Robert Alyngton, William Penbygull…). His realist metaphysical framework leads him to analyze the reference of predicates – even in the context of natural philosophy – in a way very different from the nominalist semantics of Parisian masters like John Buridan, Albert of Saxony or Marsilius of Inghen. The fact that Paul draws on the same argument in his *Expositio* as well as in his later *Summa naturalium* (1408) indicates his deep and persisting commitment to this original ontology of intensive properties.

49 Paul of Venice 1521, Metaph., c. 18, 134rb–va: "Tertia conclusio. Quaelibet intensio absoluta a qualibet qualitate realiter est distincta. […]. Quarta conclusio. Intensio absoluta est passio qualitatis intensibilis et remissibilis. Patet ista conclusio, sicut de dimensione absoluta quae est passio quantitatis continue. Haec tamen passio non est essentialis, quia potest plurificari qualitas non plurificata intensione, et econtra." See also Paul of Venice 1499, VI, tex. com. 37.

50 Paul of Venice 1521, Metaph., c. 17, 133ra, 1a concl.

His position is all the more interesting that it differs from other realist conceptions on the same problem. For instance, Duns Scotus – one of the early proponents of the addition theory, and simultaneously one of the most influential defenders of realism concerning universals – conceived the relation of a quality and its degree of intensity as a particular type of distinction, namely a 'modal distinction.' In Scotus' theory, taken up by later Scotists like Francis of Meyronnes, this phrase was meant to express the fact that the degree of a quality is not totally identical with the quality itself, even though it cannot be separated from it.[51] Paul also accepts the existence of a type of distinction weaker than the real distinction – a 'formal distinction' – which he uses like Scotus (and Wyclif, for that matter) to conceptualize the relation between essence and existence in concrete individuals as well as that between universals and particulars.[52] But Paul defines the relation between a quality and its intensity in a different way: the intensity of a quality is according to him a "proper passion" of the quality *really distinct from it*. This does not mean that intensity is a property totally external to qualities. In fact, Paul thinks that intensity is a property belonging *per se* to qualities, while the extension of a quality belongs to it *per accidens* (extension being a mode of quantity, not of quality).[53]

This view reflects a certain departure from his realist sources that is not of minor importance. Paul's ontology of accidents aims at providing a solid basis for the quantitative treatment of physical processes like motion and qualities, including alteration and intensive variations. The deep reason underlying his ontological analysis of qualities may be understood in light of his parallel understanding of motion. Paul believes that motion is a *fluxus formae*,

51 John Duns Scotus 1956, I, d. 8, p. 1, q. 3, n. 138–140; Francis of Meyronnes 1520, d. 18, q. 1, 72ra. On Scotus' position, see Cross 1998, 171–192.
52 Paul of Venice 1521, Metaph., c. 2, 118va–119va.
53 Paul of Venice 1498, I, 19vb; Paul of Venice 1521, De gen. et corr., c. 1, 48vb.

i.e. a real process on top of the mobile and its positions successively acquired.[54] He thinks that the quantitative predicates used to describe motion and its properties (speed and variations of speed) cannot have a reference if motion is reduced to the mobile and to mere relations between it and surroundings bodies. To be truly justified, the use of such predicates must be grounded on a real subject of reference, which is motion itself considered as a real process distinct from the mobile. According to Paul, just like motion is something distinct from the mobile, the intensity of a quality must be regarded as a property distinct from it. Although the arguments put forward by Paul point out the realist background underlying his ontological theory of intensive properties, the need for a semantics powerful enough to sustain the mathematical developments of his natural philosophy constitutes an additional motivation explaining his theoretical choices.

4.1.3 Graphical methods

The use of geometrical methods to illustrate the analysis of intensive properties is a remarkable feature of Paul's writings, which, serving also pedagogical purposes, is firstly grounded on ontological motivations. Indeed, the notions of degree and angle are according to Paul the two *principles* of the category of quality. In fact, Paul relies here (although without quoting him explicitly) on John Wyclif who, following a general postulate according to every genus must have a simple element as its principle, defined the notion of degree as the principle of quality.[55] Taking also into consideration qualities of the fourth species (figures), Paul supplements Wyclif's claim by adding the

54 PAUL OF VENICE 1499, III, tex. com. 4–5.

55 JOHN WYCLIF 1893, vol. 1, c. 4, 13, l. 1–10: "In omni predicamento est dare unum principium, quod est metrum et mensura omnium aliorum contentorum in illo predicamento […]; primum principium de predicamento qualitatis est gradus, quia omnis latitudo qualitatis componitur ex gradibus […]."

notion of angle as another principle of qualities. This status granted to degrees and angles explains that they possess analogical properties and justifies the appeal to geometrical illustrations. A degree is not a quality, even if, like an angle, it falls under the category of quality by reduction (*per reductionem*), just like God and prime matter are substances by reduction although they are not included in the genus of substance strictly speaking.[56]

Paul's use of geometry on the theme of intensities goes well beyond the categorical classification of degrees. Paul claims that the term "latitude", which refers to a difference between various degrees of intensity, implies a "distance" between unequal degrees.[57] The spatial character of the term is not mere metaphor here, since the intensive properties of qualities may be represented by figures. In fact, Paul's definition of what he calls uniform and difform qualities (qualities whose intensity is uniform on a surface or not, respectively) is essentially geometrical:

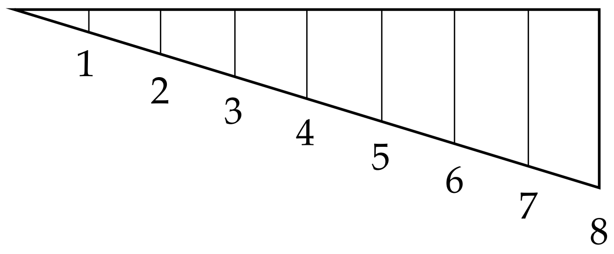
> Here it must be noted first that qualities are twofold, that is uniform or difform. A quality is uniform when all its points are equally intense, like a hotness of 4 degrees on a whole surface. But a quality is difform when its points are not equally intense, like a hotness in which one half is 4 and the other is 8. […] Second it must be noted that difform qualities are twofold, that is uniformly difform and difformly difform. A quality is uniformly difform when all its parts that are immediate extensively are also immediate intensively, or when every intrinsic degree exceeds the inferior degree precisely as much as it is exceeded by the superior one equally distant from it, like a quality imagined as a triangle. A quality is difformly difform when not all its parts that are immediate extensively are also immediate intensively, or when not every intrinsic degree exceeds the inferior degree as it is exceeded by the superior one equally distant from it, like a quality imagined as a semi-circle or a surface in which one half is one foot long and half-foot large and the other half is one foot long and equally large.[58]

56 Paul of Venice 1499, III, tex. com. 3.

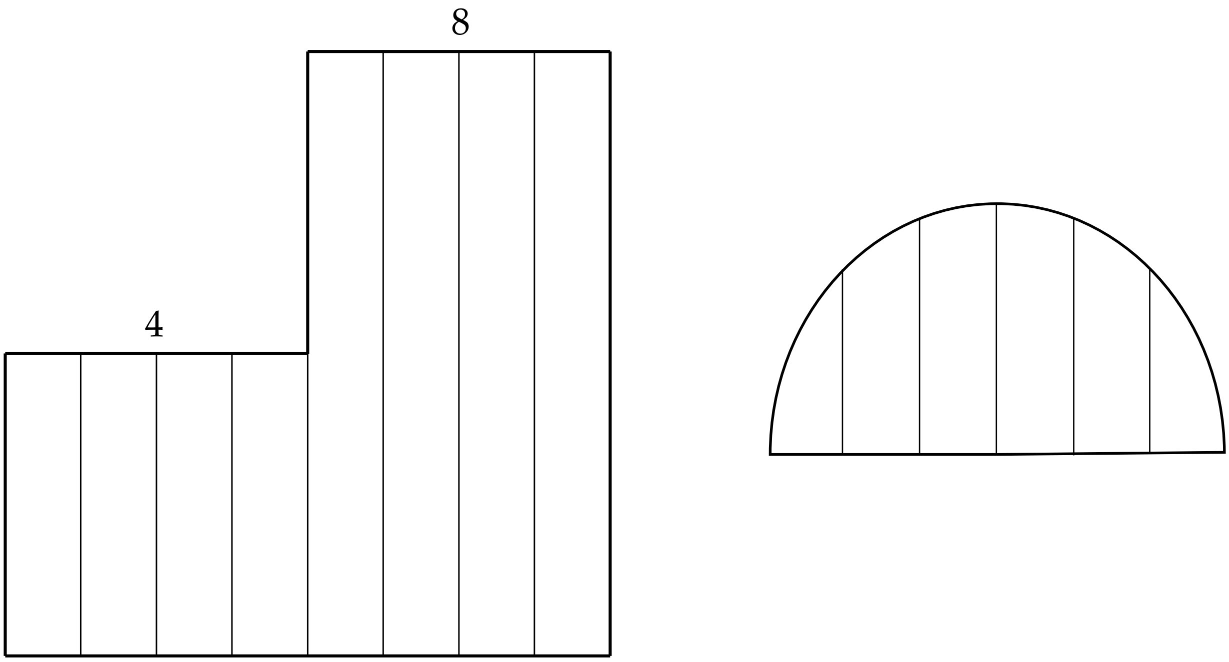
57 Paul of Venice 1521, De gen. et corr., c. 3, 49va.

58 Paul of Venice 1521, De gen. et corr, c. 3, 49va–vb: "Ubi primo notandum quod duplex est qualitas, scilicet uniformis et difformis. Qualitas uniformis est illa cuius omnia puncta sunt equaliter intensa, sicut caliditas ut quatuor per totam unam superficiem. Sed qualitas difformis est illa cuius non non omnia puncta sunt equaliter intensa, ut calidi-

Uniformly difform quality



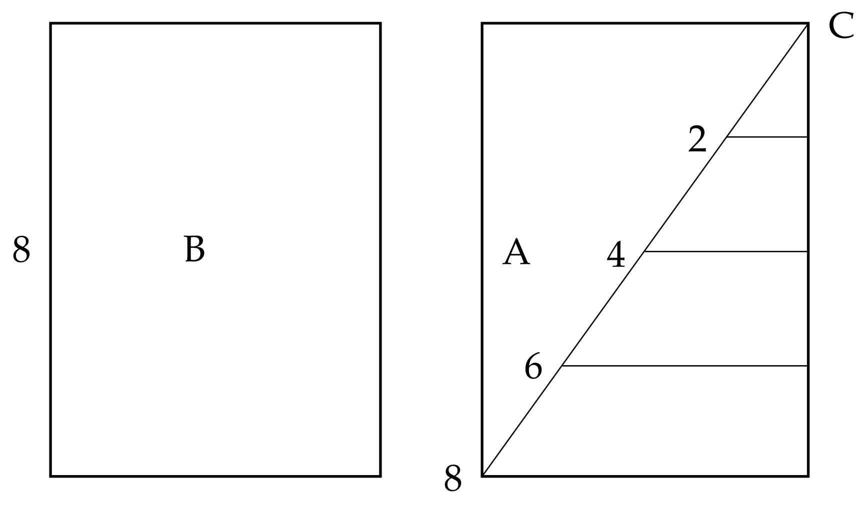
Difformly difform qualities



tas cuius una medietas est sicut quatuor et relinqua ut octo. […] Secundo notandum quod qualitas difformis est duplex, scilicet uniformiter difformis et difformiter difformis. Qualitas uniformiter difformis est illa cuius omnes partes immediate secundum extensionem sunt immediate secundum intensionem, vel cuius quilibet gradus intrinsecus tantum excedit precise inferiorem quantum exceditur a superiori equaliter distante, ut qualitas imaginata secundum formam trianguli. Qualitas difformiter difformis est illa cuius non omnes partes immediate secundum extensionem sunt immediate secundum intensionem, vel cuius non quilibet gradus intrinsecus tantum excedit inferiorem quantum exceditur a superiori equaliter distante ut qualitas imaginatur secundum figuram semicirculi aut secundum figuram superficiei cuius una medietas est pedaliter longa et semipedaliter lata, et alia pedaliter longa et pedaliter lata."

The choice of these figures is strikingly similar to the illustrations one finds in Nicole Oresme's *De configurationibus* and in the historically more influential (but far less sophisticated) *De latitudinibus formarum*, whose author's identity is still open to debate.[59]

This definition of qualitative distributions is not the only occasion for Paul to rely on geometrical devices. He also demonstrates owing to similar means that the intensive degree of a quality must be something indivisible. By representing uniform and uniformly difform qualities by figures, he shows that such figures must come into contact by a point (hence, something indivisible).[60] Paul imagines for this purpose two qualities differently distributed on a surface. The first – called B – is uniformly at degree 8. The second – called A – is uniformly difform from non-degree to degree 8. "Uniformly difform" means here that the quality has different degrees of intensity but that its variation rate is uniform (continuous). Thus, the two qualities can be represented by the rectangle B and the right-angled triangle A.



59 Nicole Oresme 1968, I, c. 14, 198–203; [Anonymous] 1486, 2a.
60 Paul of Venice 1521, Metaph., c. 17, 133va–vb.

The superposition of A and B (like on the right figure above) demonstrates that the contact point (c) between A and B does not bring anything to the intensity of B, showing that this degree is indivisible. Since the same reasoning may be applied to any degree, Paul uses this geometrical example to illustrate the fact that every degree "terminating" a latitude is indivisible.[61]

These graphical methods indicate the great diversity of sources synthetized by Paul. It is well known that among the most influential masters of the Parisian school, Albert of Saxony, John Buridan and Marsilius of Inghen made almost no use of such geometrical methods in their writings, unlike Nicole Oresme. In the Italian context, however, Giovanni Casali had also employed similar geometrical representations, which were well known and commented by the time of Paul, as evidenced by Messino da Codronchi's *Questiones super questionem Johannis de Casali*.[62] The presence of such figures in his *Summa* shows that Paul was well familiar with these tools, which he employs in accordance with the Parisian version of the addition theory that Giovanni Casali, however, did not accept.[63] It is most unlikely that Paul had direct access to copies of Oresme's *De configurationibus*, but he may have drawn his inspiration from Marsilius' commentary on Aristotle's *De generatione*, which relies on a few occasions on such representations for illustrating relations between 'degrees' and distributions of intensities.[64] The edition of Paul's *Summa naturalium* contains a diagram modeling his view that the sum of contrary degrees must be always equal to 8:[65]

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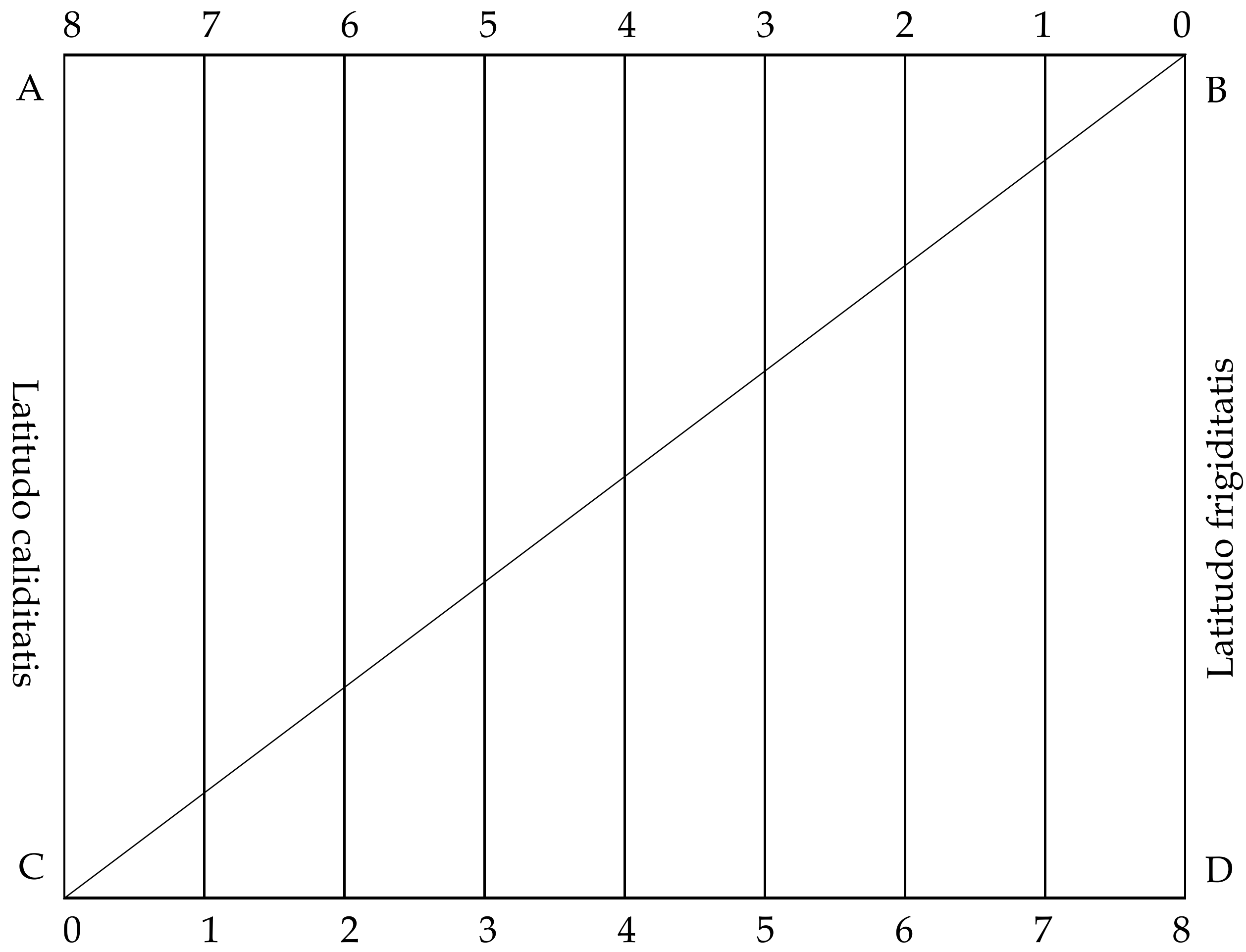
61 The same claim is made in PAUL OF VENICE 1521, De gen. et corr., c. 1, 48rb, 3a concl; PAUL OF VENICE 1521, Metaph., c. 17, 133ra.
62 JOHN CASALI 1505. On this text and its relation to Oresme's works, see CLAGETT 1959, 332; CLAGETT 1968, 66–71; SYLLA 1991, 446–453.
63 JOHN CASALI 1505, J3ra, 11a concl.
64 MARSILIUS OF INGHEN 1518, II, q. 6, a. 1, 106va.
65 PAUL OF VENICE 1521, Physic., c. 30, 24va.



Paul's geometrical illustrations cannot be compared to the much more extensive and technical functions such figures have in Oresme's and Casali's works. Still, as evidenced by his definitions of uniform and difform qualities, he does employ them to represent an extended quality's presence in a subject, i.e. not only its pure 'intensity' but also the product of intensity with extension. To summarize, Paul employs geometrical methods for analyzing qualities in two different ways. The first one is similar to Oresme's and Casali's representation techniques. It enables Paul to represent intensity at each point of a body, the bi-dimensional aspect of figures allowing him to represent both intensity and extension. Second, figures make it possible to depict the proportion of contrary degrees in a subject. In this last case, the bi-dimensionality of diagrams is meant to represent two coextended sets of contrary degrees. Such figures can represent either the *possible* values of contrary degrees in one quality or the *actual* distribution of contrary degrees on a 'linear subject' (on a line). One limitation of this method, however, is that it cannot represent the intensity of what Paul and his contemporaries call a 'difform quality' (an extended quality whose intensity is not the same everywhere) in the cases of a surface and, *a fortiori*, of a body. Indeed, in the latter cases, since the two dimensions of the figures only refer to the one-dimensional extension of contrary qualities, further dimensions cannot be represented. This limitation did not prevent Paul from dealing with cases where intensive properties are joined with more complex parameters, as his analysis of the concept of 'power' shows.

4.1.4 The dynamics of qualities: power and density

The way Paul defines the active properties of qualities equally demonstrates the direct influence of Parisian Masters, especially when it comes to the problem of reaction – a classical problem of Aristotelian philosophy. Let us recall that the problem of reaction comes from the way 'action' is characterized in the conceptual framework of Aristotelianism. The traditional Aristotelian account of natural processes explains action by the dominance of one property over another. A body warms another one if and only if its hotness is superior to the coldness of the other. But since action requires the superior intensity of a quality over another one, how can it be explained that in most cases the patient seems to react on the agent (for instance, hot water plunged into cold water is cooled by the water, but also warms it)? As S. Caroti's studies on the topic have shown, Oxfordian and Parisian Masters were strongly opposed on the way to explain such phenomena.[66] Parisian Masters, from John Buridan to Marsilius of Inghen, tried to explain the phenomenon of reaction by distinguishing two types of power within the same quality. According to this view, a quality has an active power distinct from its resistive power. Hotness, for instance is the most active quality but has a very low degree of resistive power. Even if coldness has a weaker active power than hotness, the active power of coldness is still stronger than the resistive power of hotness, which explains the interaction between these two qualities in alteration. English authors, by contrast, were reluctant to distinguish two types of powers within the same quality, and tried to account for the phenomenon of reaction on a different basis.

Like Gaetan of Thiene after him,[67] Paul totally subscribes to the Parisian theory and offers a detailed account of the respective active and resistive powers of each quality.[68] While the main representatives of the Parisian school supported this view, it was Marsilius of Inghen who systematized it and proposed in his commentary on Aristotle's *De generatione* a list of active and resistive powers of qualities ordered in complete series. Here again, Marsilius' commentary on the *De generatione* proves to be crucial for Paul's theory of qualities, which provides the same list in the same order:

> It follows from these conclusions that there is a twofold latitude in primary qualities, namely one of activity and one of resistance. The latitude of activity is

66 See esp. CAROTI 1995; CAROTI 1989.

67 GAETAN OF THIENE 1522a, 81va–83ra.

68 PAUL OF VENICE 1498, I, 46vb. Cf. PAUL OF VENICE 1499, III, tex. com. 17; PAUL OF VENICE 1521, De gen. et corr., c. 25, 62vb–63rb. Let us note that Paul distinguishes between the motive and resistive powers of the four fundamental qualities (hot, cold, wet, dry). However, when Paul more broadly divides elementary qualities into four main classes, these four qualities are referred to as "active qualities." The other three classes comprise qualities that are causes of motion (heavy and light), resistive qualities (hard and soft) and "terminative" one (diaphaneity and opacity); see PAUL OF VENICE 1498, 7ra.

> divided into four parts, of which hotness is in the first, for it is maximally active; coldness in the second, since it is less active than hotness but more than wetness and dryness; wetness in the third because it is less active than hotness and coldness but more than dryness; dryness in the fourth and smallest [*infima*] since it has the minimal activity. And the latitude of resistance is inversely divided into four parts […].[69]

Even if it appears that Paul's dynamics of properties is directly indebted to Parisian Masters, its applications suggest an equally important influence of the Oxfordian tradition. The way Paul defines the total 'power' contained in a concrete thing is typical of the Merton school and, in particular, of Richard Swineshead's *Calculationes*. In the eighth treatise of his *magnum opus*, the most mathematically refined work produced in the Calculators movement, Richard investigates the way of measuring a thing's power. By "power", Swineshead means the total quantity of a quality (heat, for instance) contained in a concrete body. He defines the "power of a thing" (*potentia rei*) as a certain magnitude composed of three parameters, among which the intensity of the quality is only one factor. The power of a thing simultaneously depends on:

1. The intensity of the quality
2. The extension of the quality in the subject
3. The density of matter in the subject.[70]

Richard Swineshead's influence on Paul is notable. Paul takes up the same concept as Swineshead for referring to this quantity: the power of a thing

69 Paul of Venice 1498, I, 49va: "Ex istis conclusionibus sequitur quod in qualitatibus primis est duplex latitudo, videlicet activitatis et resistentie. Latitudo activitatis dividitur in 4 partes in quarum prima ponitur caliditas, cum ipsa sit maxime activitatis, in secunda ponitur frigiditas, quoniam ipsa est minoris activitatis quam caliditas et maioris quam humiditas et siccitas, in tertia parte ponitur humiditas cum ipsa sit minoris activitatis quam caliditas et frigiditas et maioris quam siccitas, in quarta vero et infima ponitur siccitas, ex quo ipsa est minime activitatis. Econtra vero dividitur latitudo resistentie in 4 partes […]." Cf. Paul of Venice 1499, IV, text. com. 49.

70 Paul of Venice 1521, De gen. et corr., c. 26, 63rb–64ra.

must be measured by its "multitude of form" (*multitudo formae*), where "form" must obviously be understood in the sense of a quality. Thus, Paul denies that a thing's power only consists in the intensity of its quality, or in this intensity times its extension. The density of matter in which a quality is extended must be taken into account for quantifying the total quantity of quality contained in a body. For instance, a flame of fire has a higher degree of hotness than boiling water, but boiling water has more form – therefore more power than the flame.[71]

This definition of power leads Paul to what we could label as a 'corporeist' treatment of qualities. A body does not necessarily lose its power if it is condensed into a smaller volume, since in this case the quality contained in it is also condensed. The intensity is not affected by the density of matter insofar as an equally intensive qualitative degree can exist in differently dense bodies. Inversely, a body containing a certain degree of quality can change and acquire another volume while conserving its power, since the quality it had is condensed and, thus, preserved despite the change of extension. This is not to deny any role to extension: two equally dense bodies having the same degree of intensity but different extensions will not have the same power.

From the point of view of history of science, Paul's adoption of Swineshead's way of measuring power is important for two reasons. First, the distinction of these three parameters in the definition of a total quantity called the 'power' of a thing is crucial because, as M. Clagett rightly pointed out, it foreshadows, in the case of a quality like hotness, the modern distinction between temperature and heat.[72] Given the importance of the analysis of physical properties like heat in the Italian Renaissance, it is not surprising to

71 Paul of Venice 1498, I, 60ra. See similar remarks in Richard Swineshead 1520, VII, 27ra.
72 Clagett 1941, 58, 65.

see in the first half of the 15^{th} century authors like Gaetan of Thiene and Giovanni Marliani – that is to say the generation of thinkers succeeding Paul's – endorsing the same definition of power. But unlike Richard Swineshead, and more generally the Oxford Calculators, mostly interested in the logical and mathematical implications of physical definitions, the natural philosophers following Paul's generation will integrate the same concepts in a more empiricist approach to physical questions.

Second, the use of such a definition in a more empiricist framework entails a certain distancing from the principles of Aristotelian physics. Indeed, the Aristotelian classification of science left little room for a possibility of a mathematical physics, and the metaphysical Aristotelian framework underlying this classification prohibited the use of composite units of measurement such as speed S = Distance / Time. From an Aristotelian point of view, choosing a special magnitude composed of different categories (like quality and quantity) for a convention for measurement was problematic, if not prohibited, because of the principle of equivocity of being. The definition of 'power' as the product of three parameters overcomes this prohibition. Admittedly, no more than Swineshead, Paul does not seek a special unit (other than the phrase "multitude of form") for denominating this magnitude. But he too considers it as a physical quantity that can be measured, at least in theory, by the natural philosopher. It is on the basis of this complex (both realist and corporeist) ontology of intensive properties that Paul elaborates a finely articulated system of measurement for intensities and qualitative distributions.

4.2 System of measurement conventions

4.2.1 Single qualities

4.2.1.1 Measure of uniform qualities

In his *Summa*, Paul investigates the way to measure a body qualified in different ways, some of which are rather complex. But understanding Paul's system of measurement for complex situations where a body contains for instance different qualities requires explaining his opinion about the measure of simple qualities. How should the intensity of a quality (independently of its extension in a subject) be measured? This problem had been hotly debated since the second quarter of the 14^{th} century, especially in the Oxford Calculators school. To understand the meaning of what it meant to measure the intensity of a quality in such scholastic discussions, it should be recalled that, by the time of Paul, no unit of measure was in use for qualities like hotness, colors and the like. On the contrary, different units of measure were common for distances (e.g. foot) and weights (e.g. drachm). Although medieval discussions over the intensity of qualities usually employed the notion of 'degrees' to refer to a certain quantity of quality, no standard unit was available for most qualities. Thus, 'measuring' a quality meant evaluating its position in the scale of possible intensities ranging from non-degree to maximum degree. As mentioned above, a usual convention was to choose the number 8 for denoting the maximum degree, other frequently used numbers being 10 or 4 (especially in medical contexts for the latter). 'Measuring' qualities meant establishing a comparison between two or more qualities regarding the minimum and maximum bounds of the scale of possible degrees. In other words, one had to choose either the minimum bound or the maximum one as a reference for comparing a given degree to another. But depending on which term one chose as a reference, the value of alterations could be different. For instance, if one considers that the intensity of a quality should be measured by its distance to the non-degree (minimum limit), no quality can be said to be more than twice as intense as its mean degree, since the maximum degree is twice more distant to the minimum degree than the mean degree. On the contrary, if one chooses to say measure intensity by the reduction of the distance to the maximum degree, it is possible for a quality to be more than twice more intense than the mean degree, but one has to admit that only an *infinite* intensification would lead to the maximum degree, since the distance between a finite degree and the maximum bound, according to this convention, must be divided an infinite number of times to reach its maximum bound. The very same question could be asked for remission and, for similar reasons, different options were also available in this case.

On this problem, Paul rejects the position held by Albert of Saxony, which consists in taking the distance to minimum degree as the reference for intension and the distance to maximum degree as the reference for remission. In his *Treatise on Proportions*, Albert only dealt with the speed of alteration and did not settle the problem of how to measure qualities strictly speaking. It was only in his *Questions on the Physics* that Albert clarified his views on this point. Albert denied that measuring intension by comparison to the maximum degree would make sense, insofar as – adopting the 10 degrees scale typical of Parisian masters – one would have to say that a quality being intensified from degree 8 to degree 9 would become twice more intense, which Albert regards as a problematic consequence.[73] Albert's choice, on this basis, was to take the maximum bound only for remission and, thus, to choose different degrees of reference for the two opposite motions of alteration. It should be noted, however, that it remains uncertain whether Paul really has

73 ALBERT OF SAXONY 1999, q. 5, 966, l. 76–78.

Albert in mind when criticizing this position (even though Albert's *Questions on the Physics* enjoyed an important diffusion in Italy, they only contain a few remarks on this point).

Paul also rejects the view that both intension and remission should be measured in reference to the maximum degree. According to him, both intension and remission should be measured in reference to the non-degree.[74] On this point, Paul agrees with Richard Swineshead's view, defended in the first book of the *Calculationes*. We can note, as Paul does, that this convention is the most convenient for the aim of computing intensive degrees, since it amounts to considering remission as a mere privative phenomenon with regard to intension, without postulating a double (inverse) scale of intensive variations.[75] To summarize, Paul only agrees with Albert on the choice of the non-degree as the point of reference for intension but seems more directly influenced by Richard Swineshead with whom he entirely agrees. It is on this preliminary basis that Paul considers more complex cases of qualitative measurement.

4.2.1.2 Measurement of difform single qualities and alteration

In the *Summa naturalium*, Paul closely follows the progression of Richard Swineshead's *Calculationes*. After investigating the way of measuring the simple intensity of a quality, he adds other parameters, the first of which is extension. How must an extended quality be measured? The case is trivial for qualities extended in a subject in a uniform way, because in this case only one

74 Paul of Venice 1499, V, tex. com. 19: "Melius tamen diceretur quod intensio et remissio habet fieri per recessum a non gradu ad accessum ad non gradum et quod illud est altero intensius quod magis distat a non gradu et illud altero remissius quod magis appropinquat non gradui." See also Paul of Venice 1521, De gen. et corr., c. 2, 49ra, 4a concl.

75 Paul of Venice 1521, De gen. et corr., c. 2, 49ra: "Secundo notandum quod in qualibet latitudine qualitatis intensio et remissio se habent privative […]."

degree is present in the subject. The problem really concerns difform qualities, for which one must differentiate distinct cases.

Reciting the usual 'mean degree' or 'Merton' theorem, Paul states that a uniformly difform quality must correspond to its mean degree (a quality extended from non-degree to degree 8 corresponds to degree 4). A difformly difform quality in which the two halves are uniform corresponds to the mean degree between the two halves. For instance, if the first half of a surface is uniformly qualified at degree 8 and the other at degree 4, the total latitude of quality corresponds to degree 6.[76] The reason for this is that each part of the quality equally contributes to the "denomination" of the whole. Let us point out that Paul, unlike Richard Swineshead in the *Calculationes*, does not extend at this point this principle to proportional parts of a quality smaller than 1/2 and, more generally, does not venture into more complex cases of difformly difform distributions.[77] As we will see, nonetheless, he also accepts this principle of 'proportional denomination.'

Paul also precises how the *velocity* of alteration must be measured for the case of simple qualities (i.e. for cases where a subject acquires only one quality in a given time). The velocity of alteration must be measured according to the latitude of quality acquired by a subject, so that two subjects (equal or unequal) will be altered equally quickly if they acquire the same latitude of quality, just like augmentation is measured by the absolute quantity acquired.[78] Thus, Paul's view on this point is equivalent to his conception of the measure of quantitative change, which is, as we have seen, globally in line with Richard Swineshead's position and allows for a strictly parallel treatment of speeds of change for qualities and quantities. This observation rein-

76 PAUL OF VENICE 1521, De gen. et corr., c. 3, 50ra, 4a concl.
77 RICHARD SWINESHEAD 1520, II, 6va.
78 PAUL OF VENICE 1521, Physic., c. 37, 29vb, 4a concl.

forces the interpretation of the claim – constantly emphasized by Paul – that intensive quantities are analogous to extensive ones.[79]

4.2.2 Mixed qualities

Paul rejects the view according to which substantial forms admit of degrees.[80] Only certain types of accidents can vary in intensity. Nonetheless, he acknowledges the necessity of providing means to quantify the intensive value of elements mixed in compound bodies. Paul sustains a surprising position on the nature of mixtures. One of the most difficult problems of scholastic natural philosophy was to understand how the properties of elements can exist in a mixture if the elements themselves do not exist *in actu* in them any longer. If it is conceded that the elements still exist and remain present in the mixture, then it seems like the oneness and unified character of mixture is lost. Paul rejects Averroes' famous solution to this problem, which claimed that the substantial forms of the elements admit of degrees and persist in mixtures under a "remitted" mode of being.[81] Refusing that substantial forms can vary in intensity in this way, Paul's attempt to solve the problem relies on a distinction between two types of formal action – inhesion and information – which closely resembles the distinction between assisting and inhering forms stemming from Averroist cosmology.[82] Assisting forms, in Averroes cosmology, refer to the way the cosmic Intelligences move celestial bodies. They inform those bodies but do not inhere in them. Drawing on this distinction, John of Ripa, whose writings were familiar to Paul, had gone one step further by arguing that a form can inform matter without inhering in it and, what is more,

79 Paul of Venice 1521, Metaph., c. 18, 134ra: "Notandum primo quod intensio in qualitate se habet sicut magnitudo in quantitate."
80 Paul of Venice 1498, I, 19ra; Paul of Venice 1521, De gen. et corr., c. 10, 54rb, 1a concl.
81 Paul of Venice 1499, VI, tex. com 45.
82 On this notion, see De Libera 2014.

that a form can inform matter without constituting or informing the corresponding composite. Relying on these scholastic refinements, Paul applies a similar distinction to the problem of the elements in the mixture: elements in the mixture inform matter but do not inhere in it.[83] Evaluating the consistency of this claim would exceed the scope of this study. More relevant to our topic is the remark that this way of characterizing the elements' presence in mixture allows Paul to maintain that elemental qualities remain under a remitted mode of being in the mixture, and that elements are in a certain way still present in the mixture, although the substantial form of an element itself is not subject to intensive variation. However, since elemental qualities do admit of degrees, Paul needs to establish a convention for measuring the intensive value of elements or bodies containing different qualitative degrees.

4.2.2.1 Unequal elemental qualities

As already noted, Paul thinks that the total sum of degrees in a subject remains constant and must be equal to the maximum number of degrees attainable by one quality (8 by convention). There is in fact one exception to this rule. This rule is universally true for mixed body, but not for the elements themselves that constitute mixtures. According to Paul, single elements have only two primary qualities. In its normal state, one element has its proper quality up to the maximum degree, and its other quality to the degree immediately inferior to this one. In other words, fire has hotness up to degree 8 and dryness to degree 7, and similarly for other elements (water: coldness 8, wetness 7; air: wetness 8, hotness 7; earth: dryness 8, coldness 7).[84] In fire, dryness

83 Paul of Venice 1499, III, tex. com. 73. Cf. John of Ripa 1961, q. 1, a. 3, 6a concl., 212–213; John of Ripa 1957, q. 1, a. 2, II, 74–75. For Paul's exposition on the passage of John of Ripa's *Lectura*, see Paul of Venice 1980, 128–130.

84 Let us note than Gaetan of Thiene will rather choose the view that both elementary qualities are at their maximum degree; see his Gaetan of Thiene 1522b, 86vb.

is not mixed with any degree of wetness, although it does not have the highest possible degree. Elements are limit cases in the sense that they cannot exist by themselves, and are always mixed in compound bodies. This exception to the rule of 'latitude completion' explains that elements in the natural realm always tend to compose mixtures and to not subsist by themselves. As a consequence, even though this corresponds to their natural state, elements often happen to have their qualities at different degrees of intensity due to physical interactions. Just like in the *Calculationes*, the question of how to measure elements having two unequal qualities is the next step of Paul's reflection. How should we measure the total intensive value of a fire having hotness of degree 8 and dryness of degree 2, for example?

In the third treatise of the *Calculationes*, Richard Swineshead had defended the view that the intensive value of mixed primary qualities in a body should be measured not by the middle degree strictly equidistant from the two values but rather by their mean proportional degree (in other words, following a geometrical proportion and not a merely arithmetical excess).[85] According to Richard, an element having hotness at degree 8 and dryness at degree 2 would be qualified at degree 4, and not at degree 5. Paul rejects both solutions. The choice of the strict middle or mean proportional degree leads to physical paradoxes. The arguments provided by Paul are rather intricate and, if anything, do little justice to the position defended by Richard Swineshead. Against Richard's own view, Paul argues for instance that hotness at degree 8 and dryness at the same degree in fire denominate the subject proportionally to their compresence, so that their contribution to the whole denomination equals 4. But if dryness were remitted at degree 2, it would then only denominate the element according to 1 degree, which, ac-

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85 RICHARD SWINESHEAD 1520, III, 12rb.

cording to Paul, would make the total intensive value of the element equal to degree 5, whereas its value should rather be 4 according to the 'mean proportional degree' convention. Other arguments involve logical absurdities resulting from alteration of the two elemental qualities. Because of these consequences, elements having unequally intense qualities according to Paul should simply be measured according to the value of the most dominant quality (in the example above, the value of the element would be 8).[86]

4.2.2.2 Contrary qualities in mixtures

4.2.2.2.1 Coextended contrary qualities

The study of co-extended contrary qualities in mixtures, which is the next step in the S*umma* – corresponding to treatise 4 in the *Calculationes* – adds further complexity and, retrospectively justifies his views on the measure of non-contrary qualities in elements. Indeed, only contrary qualities reciprocally affect the denominative power of each other. Only coldness diminishes the way a certain degree of hotness qualifies a subject.[87]

The different positions that Paul rejects demonstrate once again his acquaintance with the discussions led in the *Calculationes*. The three first conventions rejected by Paul correspond to the three first positions studied by the Calculator in the corresponding treatise.[88] The first consists in measuring mixtures by the proportion of the dominant element over the weaker one. The second chooses the value of the dominant element (for example, if hotness is at degree 4 and coldness at degree 2, then the mixture is at degree 4). The third chooses a certain ratio of the strict difference between the dominant

86 PAUL OF VENICE 1521, De gen. et corr., c. 4, f. 50va, 4a concl.
87 PAUL OF VENICE 1521, De gen. et corr., c. 5, f. 50vb.
88 RICHARD SWINESHEAD 1520, IV, 12va.

element and the weaker one. In the *Calculationes*, Swineshead refuted a convention halving the difference between the two degrees (taking the previous example, the mixture would be (4 – 2) / 2 = 1 degree). Paul rejects not only this view but more generally all conventions taking any specific ratio of the difference between the two qualities (i.e. (4 – 2) / *n* in the previous example).

Against these views, Paul favors on this problem the simpler approach taking the mere difference between the contrary qualities as a convention of measurement. The reason for this is that if the intensity of the weaker quality in mixtures were increased, its role in diminishing the denominative power of the dominant quality will also increase. For instance, if coldness at degree 2 were intensified up to degree 4, the mixture will no longer be hot if it had hotness at degree 4 (because the contrary degrees will be equal). Thus, if contrary qualities contribute to their denomination of the subject proportionally to their presence in mixtures, the presence of the weaker quality must be taken into account in the measure of such bodies. Let us remark that it remains unclear how Paul's argument is supposed to support his view: his reasoning implies that different qualities denominate their subject *proportionally* to their compresence. Thus, two contrary qualities only denominate their subject according to half of their respective degree (since in this case, there are two co-extended qualities). According to him, this type of proportionality supports the choice of calculating the intensive value of the mixture by the difference between the qualities, although it is clear that alternative conventions would equally satisfy the same requirement.

4.2.2.2.2 Non-coextended contrary qualities

As we have seen, Paul admits that co-extended qualities contribute to the denomination of their subject according to half of their intrinsic degree. He takes a similar stance as to non-coextended contrary qualities. His position on this point can be deduced from two principles. The first could be labelled as 'principle of proportional denomination.' Every quality contributes to the denomination of its subject proportionally to its extension in it. Thus, half of a quality of degree 4 will denominate its subject according to 2 degrees. This principle was used by Richard Swineshead in his *Calculationes*.[89] For example, Paul deduces that a body whose halves are qualified at degree 4 and degree 6 is qualified at degree 5, each half contributing to the whole denomination to 2 and 3 degrees, respectively:[90]

A Hotness of degree 4	B Hotness of degree 6

Intensive value = Hotness of degree 5

Since Paul holds that coldness is a positive contrary of hotness, and not a mere privation, his method for measuring mixtures of non-coextended qualities requires an additional principle according to which contrary qualities prevent each other from denominating their subject. This goes for both coextended and non-coextended contraries.[91] Given these premises, Paul deduces that the intensity of a mixture having contrary qualities in different parts must be measured proportionally to their respective intensity and to their proportional extension in the subject, so that $d^{o}\mathrm{M} = d^{o}\mathrm{DQ}/n - d^{o}\mathrm{WQ}/n$, where n denotes the proportional part of the mixture M in which the dominant quality (DQ) and the weaker one (WQ) are extended. Thus, a mixture having one

89 It is in fact employed many times in the *Calculationes*. For one of the most explicit statements, see RICHARD SWINESHEAD 1520, II, 6va.

90 PAUL OF VENICE 1521, De gen. et corr., c. 6, 51rb.

91 PAUL OF VENICE 1521, De gen. et corr., c. 6, 51va.

hot half at degree 8 and another cold half at degree 4 will be hot at degree 2:[92]

A Hotness of degree 8	B Hotness of degree 4

Intensive value = Hotness of degree 2

As we can see, this convention of measurement only adds the proportionality principle following from the parameter of extension to the case of coextended contraries studied above (4.2.2.2.1). In his *Abbreviation* of John of Ripa's *Lectura super Sententiarum* (1401), Paul discusses the same problem and evaluates in much more detail the *pro et contra* arguments for the view that a quality denominates the whole subject in which it inheres proportionally to its extension.[93] The structure and arguments of Paul's *Summa* dedicated to the methods for measuring qualitative intensities show that he most likely had Richard Swineshead's *Calculationes* before his eyes when he composed this passage. That said, he does not faithfully repeat Richard's view but designs arguments of his own when it comes to mixed and extended qualities.

5. Concluding remarks

A comparative survey of Paul's opinions regarding the measurement methods for natural properties shows that he equally borrows from Parisian and Oxfordian sources, as shown in the diagram below (which, however, does not take into account the various cases of qualitative distributions on which Paul's views sometimes depart from Swineshead's):

92 PAUL OF VENICE 1521, De gen. et corr., c. 6, 51va, 4a concl.
93 PAUL OF VENICE 2000, d. 17, q. 4, a. 2, 395–398.

	Local motion ***tanquam penes causam***	**Local motion** ***tanquam penes effectum***	**Quantitative motion**	**Qualities**
Similar to	Both Oxfordian tradition & Parisian school	Albert of Saxony	Roger Swineshead (& Richard Swineshead)	Richard Swineshead (for simple qualities)

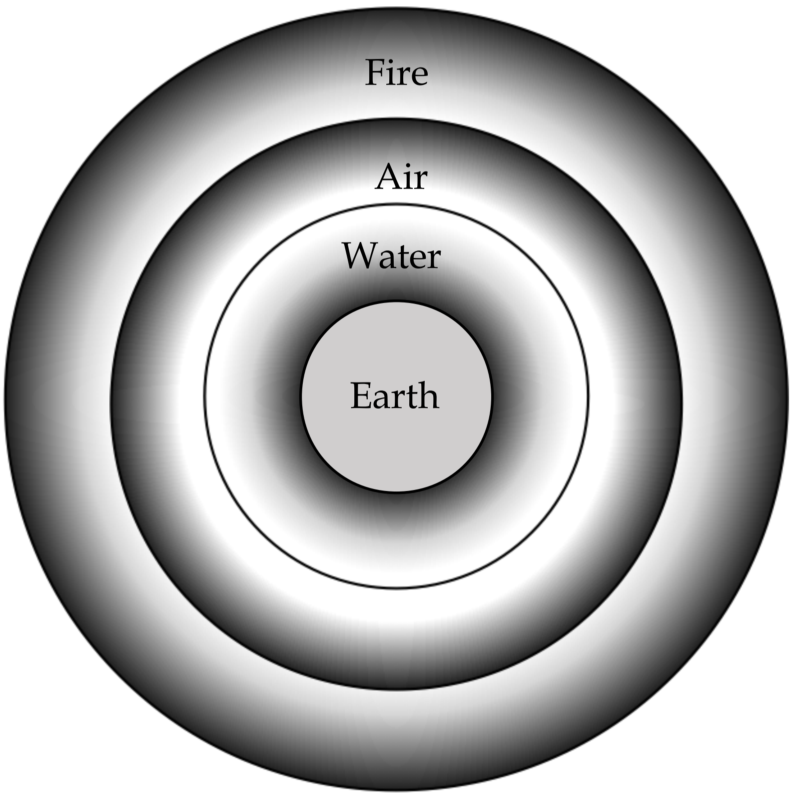
Combining various doctrinal influences, Paul conceived an original ontology of intensive properties, characterized by realist commitments that mark him out from his predecessors. Even when he seems to merely paraphrase Albert of Saxony, like on the measure of local motion, his underlying account of motion that he conceived as a *fluxus formae*, i.e. as a successive entity really added to the states acquired by the mobile, testifies to the synthetic character of his construction.

It would require a special study to evaluate Paul's claim that the intensity of a quality is really distinct from it, the consistency of which may be called into question (let us note that he cautiously refrains from making the same claim about speed and its intensity). Be that as it may, this theory enables him to offer a more satisfactory account – to some extent – of the structure of properties than nominalist views. By claiming that the degree of a quality is a property really distinct from it, Paul has the means to explain why and how a quality may vary in intensity while remaining the same from the point of view of its species or kind. In this way, Paul can maintain the existence of real classes of natural properties, some of which are structured according to real relations of contrariety despite changes in the proportion of qualitative degrees.[94] This ontology of qualities is in line with his metaphysical analyses of the perfection of species, which he also considers as quantifiable and measurable. The use of geometrical methods, grounded on a certain conception of degrees and angles as principles of qualities, will prove equally important for his view on this problem proper to late medieval metaphysics, although this point also exceeds the scope of this study. From these observations, still, we can already conclude that Paul sought to base his physics of natural properties on a robust realist ontology inspired not only by Wyclif and Burley but also other sources like John of Ripa, and powerful enough to make it possible to extend his measurement framework to metaphysical problems.

An interesting consequence of his hybrid doctrinal construction is that, far from preventing him to apply these logical and mathematical tools to empirical problems, Paul seems to have envisioned a quite broad use of them. For instance, he uses those analytical devices in his cosmological reflections. Paul states that the highest spheres containing one element (i.e. the elemental spheres) start from their highest degree to their most remitted degree at the point where they reach the limit of the inferior element. Here, the influence of the Calculatory tradition is visible from a twofold point of view. First, Paul applies the concepts of "uniformity," "difformity" and "uniform difformity" – originally used to describe distributions of qualities or speeds – to these elemental spheres. The distribution from the highest degree to the least intense one is according to Paul "uniformly difform." Thus, the most intense degree of fire can be found at the outer limit right below the sphere of the Moon, whereas its most remitted degree is located at the beginning of the (inferior) sphere of air. Between those two extremes, fire presents a uniformly difform

94 Paul presents himself as a realist about relations against Ockham, Buridan and Gregory of Rimini; cf. PAUL OF VENICE 1499, V, tex. com. 10.

distribution. Second, Paul uses the technical terms "extrinsic" and "intrinsic limit", typical of the logical tools inherited from the Calculators, to describe the point where these extreme degrees are precisely located.[95] The virtues of fire and air start *inclusively* at the limit of their superior sphere and are terminated *exclusively* at their inferior sphere. Thus, they have their most intense degrees at the outer limit of the superior sphere, and their most remitted degree at the inferior one. On the contrary, water is terminated *inclusively* at the inferior sphere (earth) in such a way that its most intense degree is located at this inferior limit, whereas the less intense degree of water is located at the limit of its superior sphere (air), where water ends exclusively. The qualitative distribution of the virtue of water is also uniformly difform, but it is inverse to that of fire and air, which explains that water is *mixed* with earth at least in the most superficial part of it (e.g. oceans, rivers and so on):



95 Paul of Venice 1499, IV, tex. com. 49–50.

It remains unclear, admittedly, how Paul's distinction between inclusive and exclusive limits is supposed to really *explain* the intermingling of water and earth that we observe at the surface of the earth. But the interest of Paul's explanation lies elsewhere. Its originality comes from the adaptation of conceptual devices originally meant for logical and sophismatic purposes to cosmological enigmas. Here, Paul demonstrates a will to use such conceptual tools to more empirical and concrete problems. Other empirical applications of these analytical methods can be seen when Paul recalls (even if rather vaguely) his convention for measuring local motion *tanquam penes effectum* when discussing astronomical issues,[96] or for explaining the spontaneous cooling of hot water in a container (problems that will be intensely discussed in the Italian context by authors like Giovanni Arcolani or Giovanni Marliani).[97] In this respect, Paul is a prime example of a trend proper to Italian thinkers of the late 14th century, who undertook to detach analytical techniques from purely logical contexts and to apply them to more and more empirical issues, like Peter of Mantua who in the same years extends conceptual analyses *de incipit et desinit* to a striking number of questions of natural philosophy.[98] It is no accident that the same approach will be a central characteristic of Paul's student Gaetan of Thiene who, relying more importantly on William Heytesbury, will make a similar use of the *Calculatores* innovations in a considerable number of empirical problems.

From these observations, and given his central influence in the north Italian context of the late 14th century and early 15th century (later sustained by the many editions of his works), it can be established that Paul played an important role in the transition from the Aristotelian qualitative natural

96 PAUL OF VENICE 1521, De celi et mundi, c. 15, 42rb, ad 2m.
97 CLAGETT 1941, 61–64, 67.
98 See JAMES 1968.

philosophy to the modern mathematical conception of physics.[99] True, Paul did not invent new theories of mountains, rainbows or astronomical bodies. Nor did he contribute to the development of mathematics *per se*. But he was nonetheless a central actor in the transmission of a new way of doing physics, by applying conceptual techniques invented in the 14th century to a broader range of physical questions. What is more, his importance cannot be limited to this role of transmission. Paul elaborated a finely articulated approach to quantification, consistently grounded on exceptionally strong metaphysical assumptions, which enabled him not only to provide a theory of measurement encompassing both physical and metaphysical properties, but also to apply those techniques to several empirical subjects. In light of the rise and development of modern scientific thought in the late Middle Ages, it is hoped that this study will help better appreciate Paul's role and influence on the Italian intellectual context of the 15th century.

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99 See the parallel remarks of WALLACE 1972, vol 1, 121–127.

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